Think Academy

Day 1

Review

1. If $|a-1|=a-1$, what is the range of a?

A. $a\geqslant 1$ B. $a\leqslant 1$ C. $a<1$ D. $a>1$

2. If $3<a<10$, $|3-a|+|a-10|=$ ______ .

3. When $a<-2$, simplify $|1-a|+|2a+1|+|a|$.

4. Suppose $\sqrt{a^2-b}+|b^3-343|=0$, $ab=$ ______ .

5. If a, b, c are all integers, and $|a-b|^{19}+|c-a|^{99}=1$, $|c-a|+|b-c|+|a-b|=$ ______ .

Preview

6 Solve the absolute-value inequalities.

(1) $|x-5|+|x|+|x+2|<15$.

(2) $|1-x|+|x-2|>x+3$.

7 For $|x|+|x-1|+|x-2|+|x-3|+\cdots+|x-100|$, the minimum value is ______ .

Day 2

Review

1. Which of the following is the factorization?

A. $ab+a+1=a(b+1)+1$

B. $y^3+3y^2=y^2(y+3)$

C. $x^2+1=x\left(x+\frac{1}{x}\right)$

D. $m(a+b-c)=ma+mb-mc$

2. The polynomial: x^3+ax can be factored as $x\left(x-\frac{1}{2}\right)(x+b)$. Find the value of a and b.

3. The integers x and y satisfy the equation: $2xy+x+y=83$, $x+y=$ ______ .

4. Given $(x^2+y^2)^4-6(x^2+y^2)^2+9=0$ with respect with x and y, $x^2+y^2=$ ______ .

A. $\sqrt{2}$ B. $\sqrt{3}$ C. 2 D. $\sqrt{5}$

5. For any integer m, $(4m+5)^2-9$ must can be ( ) .

A. divided by 8

B. divided by m

C. divided by $m-91$

D. divided by $2m-1$

Preview

6 Factoriazation: $(x+1)^4+(x^2-1)^2+(x-1)^4$.

7 Factorization: $4\left(3x^2-x-1\right)\left(x^2+2x-3\right)-\left(4x^2+x-4\right)^2$.

Think Academy

Day 3

Review

1 Given that $\{a_n\}$ is an arithmetic sequence, if $a_{10}=1$, $a_{15}=3$, $d=$ ______ , $a_{20}=$ ______ , $a_{30}=$ ______ .

2 The sum of the first n terms of an arithmetic sequence $\{a_n\}$ is denoted by S_n. If $S_2=2$ and $S_4=8$, then the value of S_6 is ______ .

3 The arithmetic sequence $\{a_n\}$ is given, where $a_5=33$ and $a_{45}=153$. The term 201 is the ( ) term of the sequence.

A. 60 B. 61 C. 62 D. 63

4 For the geometric sequence: $\{a_n\}$, $a_2=4$, $a_3=8$, what is the value of $a_1+a_2+a_3+a_4$?

A. 30 B. 28 C. 24 D. 15

5

In a geometric sequence $\{a_n\}$ where all terms are positive, $a_1 = 3$ and the sum of the first 3 terms is 21. Therefore, what is the value of $a_3 + a_4 + a_5$?

A. 33 B. 72 C. 84 D. 189

6 For the geometric sequence: $\{a_n\}$, common ratio is $q = 2$, $a_2 + a_4 + a_6 + \cdots + a_{100} = 100$, what is the value of $a_1 + a_3 + a_5 + \cdots + a_{99}$?

A. 10 B. 25 C. 50 D. 200

Preview

7 The sequence: $\{a_n\}$ satisfies $a_1 = 1$, and $a_n + a_{n+1} = 3n + 2$ is true, for any $n \in \mathbf{N}^*$, $a_{2020} =$ ______ .

8 If the sequence $\{b_n\}$ satisfies the equation $\frac{b_1}{3} + \frac{b_2}{3^2} + \cdots + \frac{b_n}{3^n} = 3n + 3$ for $n \in \mathbb{N}_+$, then the sum of the first n terms of the sequence, S_n, is ______ .

Day 4

Review

1 Given the complex numbers: $z_1 = 1 + \mathrm{i}$ and $z_2 = x + 2\mathrm{i}$ ($x \in \mathbf{R}$) , if the result of $z_1 \cdot z_2$ is a real number, what is the value of x?

A. -2 B. -1 C. 1 D. 2

2 Calculate: $\frac{(2+\mathrm{i})^2}{3-4\mathrm{i}} =$ ______ .

3 Given the complex number: $z = \frac{m^2 - m - 6}{m+3} + (m^2 - 2m - 15)\mathrm{i}$:

(1) When z is a real number, find the value of real number m.

(2) When z is a complex number, find the value of real number m.

(3) When z is a imaginary number, find the value of real number m.

4 On the complex plane, point A represents the complex number:

$z=(x^2-2x-3)+(x^2+3x+2)\mathrm{i}$.

(1) When z is a real number, find the value of real number x.

(2) If point A is belong to Quadrant II, find the range of x.

5 On the complex plane, which Quadrant the point represents the complex number $\frac{\mathrm{i}}{1-\mathrm{i}}$ belongs to?

A. Quadrant I B. Quadrant II C. Quadrant III D. Quadrant IV

Preview

6 Given that complex number z satisfies $|z-1|=1$, what is the maximum value of $|2z-2-4\mathrm{i}|$?

A. 5 B. 6 C. 7 D. 8

7 Assume m and $n \in \mathbf{R}$, two roots of the equation $x^2+mx+n=0$(respects with x) are α and β, respectively.

(1) When $\alpha=1+\mathrm{i}$, find the value of β, m , $and\ n$.

(2) When $m=2$, $n=4$, find the value of $|\alpha|+|\beta|$.

Day 5

Review

1 Solve the quadratic equations.

(1) $3x^2+2x-5=0$

(2) $x(x-1)=2-2x$

2 If the equation $(m-2)x^2-2x+1=0$ has two distinct real roots, what is the range of m?

A. $m<3$ B. $m<3$ and $m\neq 2$ C. $m\leqslant 3$ D. $m\leqslant 3$ and $m\neq 2$

3 Given the function: $f(x)=(m^2+2m)\cdot x^{m^2+m-1}$.

(1) If $f(x)$ is a proportional function, what is the value of real number m?

(2) If $f(x)$ is a inverse proportional function, what is the value of real number m?

(3) If $f(x)$ is a quadratic function, what is the value of real number m?

4 What are the maximum and minimum values of the function $y=-x^2+4x+5$, where $x \in [1,4]$, respectively?

A. 8, 9 B. 5, 9 C. 5, 8 D. 1, 8

5 Given the function $f(x)=x^2-2kx+2$, it is known that for $x \geqslant -1$, and $f(x) \geqslant k$. Find the range of real numbers for k.

Preview

6 Given the inequality:$ax^2+3x+2>0(a \in \mathbf{R})$, with respect to x.

(1) The solution set of the inequality $ax^2+3x+2>0$ is given by $\{x|b<x<1\}$. Find the values of a and b.

(2) The solution set of the inequality $ax^2+3x+2>-ax-1$ (where $a>0$) is as follows.

7 Given the function: $f(x)=\begin{cases}-x^2+2x, x\leqslant a\\ x, x>a\end{cases}$.

(1) When $a=1$, the range of $f(x)$ is ______ .

(2) If the graph of the function $f(x)$ has only one common point with the line $y=a$, the range of the real number a is ______ (to be determined).

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Day 6

Review

1. Given seven cards labeled with numbers -3, -2, -1, 0, 1, 2, 3, all having the same backside, the probability of drawing a card with an absolute value less than 2 is ______ .

2. In an opaque bag, there are 3 red balls and 1 yellow ball. They only differ in color. Two balls are randomly drawn from the bag. What is the probability of drawing exactly one yellow ball and one red ball?

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{6}$

3. The probability of selecting student A from a group of 5 students, including A and B, at random to choose 2 people is ( ) .

A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{8}{25}$ D. $\frac{9}{25}$

4. In a box, there are 10 ping pong balls, including 6 new balls and 4 old balls. Two balls are randomly drawn without replacement for use. Given that a new ball is drawn on the first draw, what is the probability of drawing another new ball on the second draw?

A. $\frac{3}{5}$ B. $\frac{1}{10}$ C. $\frac{5}{9}$ D. $\frac{2}{5}$

5. When tossing a fair coin three times in succession, what is the probability of getting at least one head?

A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$

Preview

6 Put 7 same balls into 4 different boxes, marked as A, B, C, and D, respectively. Each box has 1 ball, at least. What is the probability of there are 3 balls in box A?

A. $\frac{3}{10}$ B. $\frac{2}{5}$ C. $\frac{3}{20}$ D. $\frac{1}{4}$

7 In an opaque cloth bag, there are a total of 200 balls, consisting of red, yellow, and blue balls. Except for their colors, all the balls are identical. After conducting multiple experiments of drawing balls, a student observed that the frequency of drawing red balls stabilized at 35% and the frequency of drawing blue balls stabilized at 55%. Therefore, the number of possible yellow balls in the bag is ______ .

Day 7

Review

1. ${}_5C_5 +{}_6C_5 +{}_7C_5 +{}_8C_5 = (\quad)$.

A. 28　　B. 126　　C. 84　　D. 70

2. Selecting 2 students randomly from a group of 5 students, A, B, C, D, and E. What is the probability of selecting A?

A. $\frac{1}{5}$　　B. $\frac{2}{5}$　　C. $\frac{8}{25}$　　D. $\frac{9}{25}$

3. Companies A, B, and C are bidding for 9 projects. Company A bids for 3 projects, Company B bids for 2 projects, and Company C bids for 4 projects. How many different bidding combinations are there in total?

4 The four roommates each write a greeting card. The cards are then collected and redistributed so that each person receives a card from someone else. The number of different ways to distribute the four greeting cards is ______ .

5 There are 6 numbers: 0, 1, 2, 3, 4, 5.

(1) How many odd three-digit numbers can be formed without repeating any digits?

(2) How many natural numbers less than 1000 can be formed with non-repeating digits?

Preview

6 There are ______ different ways to arrange 6 students in a row, where student A is adjacent to student B and student A is not adjacent to student C.

7 There are a total of 16 different cards, including 4 red, 4 yellow, 4 blue, and 4 green cards. We want to select 3 cards, satisfying the condition that the 3 cards cannot be of the same color, and there can be at most 1 red card. How many different ways to select the cards are there?

A. 232 B. 252 C. 472 D. 484

Day 1

Review

1 If $|a-1|=a-1$, what is the range of a?

A. $a \geqslant 1$　　B. $a \leqslant 1$　　C. $a<1$　　D. $a>1$

Answer A

Solution $\because |a-1|=a-1$,

$\therefore a-1 \geqslant 0$,

so $a \geqslant 1$.

2 If $3<a<10$, $|3-a|+|a-10|=$ ______ .

Answer 7

Solution $\because 3<a<10$

$\therefore 3-a<0$, $a-10<0$

$\therefore |3-a|+|a-10|$

$=-(3-a)-(a-10)$

$=-3+a-a+10$

$=7$.

3 When $a<-2$, simplify $|1-a|+|2a+1|+|a|$.

Answer $-4a$

Solution $\because a<-2$

$\therefore 1-a>0$, $2a+1<0$

so $|1-a|=1-a$, $|2a+1|=-(2a+1)$, $|a|=-a$

$\therefore |1-a|+|2a+1|+|a|=1-a-(2a+1)-a=-4a$.

4 Suppose $\sqrt{a^2-b}+|b^3-343|=0$, $ab=$ ______ .

Answer $\pm 7\sqrt{7}$

Solution $\because \sqrt{a^2-b}+|b^3-343|=0$

$\therefore b^3-343=0$, $a^2-b=0$

$b=7$, $a^2=b$, $a=\pm\sqrt{7}$

$ab=\pm 7\sqrt{7}$

5 If a, b, c are all integers, and $|a-b|^{19}+|c-a|^{99}=1$, $|c-a|+|b-c|+|a-b|=$ ______ .

Answer 2

Solution $|a-b|^{19}+|c-a|^{99}=1$,

$\because a$, b, c are intergers,

$\therefore$ the value of $a-b$ and $c-a$ are integers,

and $|a-b|^{19}$, $|c-a|^{99}$ are $\geqslant 0$,

the sum of 2 non-negative intergers is 1, so one of them is equal to 1, another one is equal to 0.

①$|a-b|^{19}=1$, $|c-a|^{99}=0$,

$\therefore a-b=1$, $a=c$,

so $c-b=1$;

②$|a-b|^{19}=0$, $|c-a|^{99}=1$,

$\therefore a=b$, $c-a=1$,

so $c-b=1$,

For ①, $|c-a|+|a-b|+|b-c|$

$=0+1+1$

$=2$;

For ②, $|c-a|+|a-b|+|b-c|$

$=1+0+1$

$=2$,

So the answer is 2.

Preview

6 Solve the absolute-value inequalities.

(1) $|x-5|+|x|+|x+2|<15.$

(2) $|1-x|+|x-2|>x+3.$

Answer (1) $-4<x<6.$

(2) $x<0$ or $x>6.$

Solution (1) The meaning of the set on the left side of the inequality is the sum of the distances from the points x, 5, 0, and -2. When $x=0$, the set on the left side achieves its minimum value.

When $x\leqslant -2$, the inequality can be simplified as $5-x-x-x-2<15$, which gives $x>-4$. Therefore, the inequality holds for $-4<x\leqslant -2$.

When $-2<x\leqslant 0$, the inequality can be simplified as $5-x-x+x+2<15$, which gives $x>-8$. Therefore, the inequality holds for $-2<x\leqslant 0$.

When $0<x\leqslant 5$, the inequality can be simplified as $5-x+x+x+2<15$, which gives $x<8$. Therefore, the inequality holds for $0<x\leqslant 5$.

When $x>5$, the inequality can be simplified as $x-5+x+x+2<15$, which gives $x<6$. Therefore, the inequality holds for $5<x<6$.

In conclusion, the inequality holds for $-4<x<6$.

(2) Assuming that $x\leqslant 1$, the left side of the equation can be transformed as follows: $1-x+2-x=3-2x$.

$3-2x>x+3$, which implies $3x<0$ and $x<0$. Therefore, $x<0$.

Assuming that $x\geqslant 2$, the left side of the equation can be transformed as follows: $x-1+x-2=2x-3$.

$2x-3>x+3$, which implies $x>6$. Therefore, $x>6$.

Assuming that $1<x<2$, the left side of the equation can be transformed as follows: $x-1+2-x=1$.

$1>x+3$, which implies $x<-2$.

This assumption contradicts the previous assumptions.

In conclusion, the correct solution is $x<0$ or $x>6$.

7 For $|x|+|x-1|+|x-2|+|x-3|+\cdots+|x-100|$, the minimum value is ______ .

Answer 2550

Solution When $x=50$, the minimum value is $(1+2+3+\cdots+50)\times 2=50\times 51=2550$.

Day 2

Review

1 Which of the following is the factorization?

A. $ab+a+1=a(b+1)+1$

B. $y^3+3y^2=y^2(y+3)$

C. $x^2+1=x\left(x+\frac{1}{x}\right)$

D. $m(a+b-c)=ma+mb-mc$

Answer B

Solution The answer is option B. Option A is not in product form. Option C cannot be factored without fractions. Option D involves polynomial multiplication, not factoring.

2 The polynomial: x^3+ax can be factored as $x\left(x-\frac{1}{2}\right)(x+b)$. Find the value of a and b.

Answer $a=-\frac{1}{4}$, $b=\frac{1}{2}$.

Solution $x^3+ax=x(x^2+a)$,

so $(x^2+a)=\left(x-\frac{1}{2}\right)(x+b)$,

According to $-a=\frac{1}{2}b$, and $b=\frac{1}{2}$,

so $a=-\frac{1}{4}$, $b=\frac{1}{2}$.

3 The integers x and y satisfy the equation: $2xy+x+y=83$, $x+y=$ ______ .

Answer 83 or -85

Solution $4xy+2x+2y+1=166+1$, so $(2x+1)(2y+1)=167$.

4 Given $(x^2+y^2)^4-6(x^2+y^2)^2+9=0$ with respect with x and y, $x^2+y^2=$ ______ .

A. $\sqrt{2}$ B. $\sqrt{3}$ C. 2 D. $\sqrt{5}$

Answer B

Solution Assume $t=(x^2+y^2)^2$, so $t^2-6t+9=0$

$(t-3)^2=0$, $t=3$

$\therefore (x^2+y^2)^2=3$

$\therefore x^2+y^2=\sqrt{3}$

5 For any integer m, $(4m+5)^2-9$ must can be ( ) .

A. divided by 8

B. divided by m

C. divided by $m-91$

D. divided by $2m-1$

Answer A

Solution $(4m+5)^2-9$

$= (4m + 5 + 3)(4m + 5 - 3)$

$= (4m + 8)(4m + 2)$

$= 8(m + 2)(2m + 1)$

$\therefore so$ $(4m + 5)^2 - 9$ must can be divided by 8, $m + 2$, and $2m + 1$,

so B, C, and D are wrong.

so A.

Preview

6 Factoriazation: $(x+1)^4 + (x^2-1)^2 + (x-1)^4$.

Answer $(3x^2+1)(x^2+3)$.

Solution Method 1: extend.

$= (x^2+2x+1)^2 + (x^4-2x^2+1) + (x^2-2x+1)^2 = 3x^4 + 10x^2 + 3$

$= (3x^2+1)(x^2+3)$.

Method 2: difference between square pattern.

$= (x+1)^4 + 2(x^2-1)^2 + (x-1)^4 - (x^2-1)^2$

$= (2x^2+2)^2 - (x^2-1)^2 = (3x^2+1)(x^2+3)$.

7 Factorization: $4\left(3x^2-x-1\right)\left(x^2+2x-3\right) - \left(4x^2+x-4\right)^2$.

Answer $-(2x^2-3x+2)^2$.

Solution $(3x^2-x-1)+(x^2+2x-3)=4x^2+x-4$,

Assume $3x^2-x-1=A$, $x^2+2x-3=B$,

$4x^2+x-4=A+B$.

$=4AB-(A+B)^2$

$=-A^2-B^2+2AB$

$=-(A-B)^2$

$=-\left[(3x^2-x-1)-(x^2+2x-3)\right]^2$

$=-(2x^2-3x+2)^2$.

Day 3

Review

1 Given that $\{a_n\}$ is an arithmetic sequence, if $a_{10}=1$, $a_{15}=3$, $d=$ ______ , $a_{20}=$ ______ ,

$a_{30}=$ ______ .

Answer 1:$\frac{2}{5}$

2:5

3:9

Solution $a_{20}-a_{15}=a_{15}-a_{10}$, so $a_{20}=5$;

a_{10}, a_{20}, a_{30} are Arithmetic sequences, so $a_{30}=9$. $d=\frac{a_{15}-a_{10}}{15-10}=\frac{2}{5}$.

2 The sum of the first n terms of an arithmetic sequence $\{a_n\}$ is denoted by S_n. If $S_2=2$ and $S_4=8$, then the value of S_6 is ______ .

Answer 18

Solution By the properties of the sum of the first n terms of an arithmetic sequence $\{a_n\}$, we can deduce that S_2, $S_4 - S_2$, and $S_6 - S_4$ form an arithmetic sequence.

Therefore, solving for S_6, we find that $S_6 = 18$.

Hence, the answer is 18.

3 The arithmetic sequence $\{a_n\}$ is given, where $a_5 = 33$ and $a_{45} = 153$. The term 201 is the ( ) term of the sequence.

A. 60 B. 61 C. 62 D. 63

Answer B

Solution $\because \{a_n\}$ is the Arithmetic sequence ,

$\because a_5 = 33,\ a_{45} = 153,$

$\therefore d = 3,$

so $a_n = a_{45} + 3(n - 45),$

when $a_n = 153 + 3(n - 45) = 201,$

$n = 61.$

4 For the geometric sequence: $\{a_n\}$, $a_2 = 4$, $a_3 = 8$, what is the value of $a_1 + a_2 + a_3 + a_4$?

A. 30 B. 28 C. 24 D. 15

Answer A

Solution $a_1 = 2$, $a_4 = 16$, so $a_1 + a_2 + a_3 + a_4 = 30$.

5 In a geometric sequence $\{a_n\}$ where all terms are positive, $a_1 = 3$ and the sum of the first 3 terms is 21. Therefore, what is the value of $a_3 + a_4 + a_5$?

A. 33 B. 72 C. 84 D. 189

Answer C

Solution In a geometric sequence $\{a_n\}$ where all terms are positive,

$a_1=3$ and $a_1+a_2+a_3=21$,

Hence, $q+q^2-6=0$,

Therefore, $(q-2)(q+3)=0$,

Solving for q, we get $q=2$ or $q=-3$ (discarded),

Thus, $a_3+a_4+a_5=a_1(q^2+q^3+q^4)=3(4+8+16)=84$.

Hence, the answer is C.

6 For the geometric sequence: $\{a_n\}$, common ratio is $q=2$, $a_2+a_4+a_6+\cdots+a_{100}=100$, what is the value of $a_1+a_3+a_5+\cdots+a_{99}$?

A. 10 B. 25 C. 50 D. 200

Answer C

Solution $\because\{a_n\}$ is a geometric sequence, and $a_2+a_4+a_6+\cdots+a_{100}=100$,

$\therefore a_1q+a_3q+\cdots a_{99}q=100$,

$q=2$, $\therefore a_1+a_3+a_5+\cdots+a_{99}=50$.

Preview

7 The sequence: $\{a_n\}$ satisfies $a_1=1$, and $a_n+a_{n+1}=3n+2$ is true, for any $n\in\mathbf{N}^*$, $a_{2020}=$ ______ .

Answer 3031

Solution For any $n \in \mathbb{N}^*$, the equation $a_n + a_{n+1} = 3n + 2$ holds.

When $n \geqslant 2$, we can obtain: $a_{n-1} + a_n = 3n - 1$.

Subtracting the two equations, we have: $a_{n+1} - a_{n-1} = 3$.

When $n = 1$, we have: $a_1 + a_2 = 5$, which gives us $a_2 = 4$.

Therefore, the sequence $\{a_{2n}\}$ forms an arithmetic progression with first term 4 and common difference 3.

Hence, $a_{2020} = 4 + 3 \times 1009 = 3031$.

Therefore, the answer is 3031.

8 If the sequence $\{b_n\}$ satisfies the equation $\frac{b_1}{3} + \frac{b_2}{3^2} + \cdots + \frac{b_n}{3^n} = 3n + 3$ for $n \in \mathbb{N}_+$, then the sum of the first n terms of the sequence, S_n, is ______ .

Answer $\frac{3^{n+2} + 9}{2}$

Solution Because $\frac{b_1}{3} + \frac{b_2}{3^2} + \cdots + \frac{b_{n-1}}{3^{n-1}} + \frac{b_n}{3^n} = 3n + 3$ (1),

So when $n \geqslant 2$, $\frac{b_1}{3} + \frac{b_2}{3^2} + \cdots + \frac{b_{n-1}}{3^{n-1}} = 3(n-1) + 3 = 3n$ (2),

By subtracting (2) from (1), we get $\frac{b_n}{3^n} = 3$,

So when $n \geqslant 2$, $b_n = 3^{n+1}$,

When $n = 1$, $\frac{b_1}{3} = 6$, i.e., $b_1 = 18$, which does not satisfy the given equation,

Therefore, the general term of the sequence $\{b_n\}$ is $b_n = \begin{cases} 18, & n = 1 \\ 3^{n+1}, & n \geqslant 2 \end{cases}$,

Thus, the sum of the first n terms of the sequence $\{b_n\}$ is

$S_n = 18 + \frac{3^3(1 - 3^{n-1})}{1 - 3} = \frac{3^{n+2} + 9}{2}$.

Hence, the answer is $\dfrac{3^{n+2}+9}{2}$.

Day 4

Review

1 Given the complex numbers: $z_1 = 1 + \mathrm{i}$ and $z_2 = x + 2\mathrm{i}$ ($x \in \mathbf{R}$) , if the result of $z_1 \cdot z_2$ is a real number, what is the value of x?

A. -2 B. -1 C. 1 D. 2

Answer A

Solution $z_1 \cdot z_2 = (1+\mathrm{i}) \cdot (x + 2\mathrm{i}) = (x-2) + (2+x)\mathrm{i}$, $\because z_1 \cdot z_2 \in \mathbf{R}$, $\therefore 2 + x = 0$, $\therefore x = -2$.

2 Calculate: $\dfrac{(2+\mathrm{i})^2}{3-4\mathrm{i}} =$ ______ .

Answer $-\dfrac{7}{25} + \dfrac{24}{25}\mathrm{i}$

Solution $\dfrac{(2+\mathrm{i})^2}{3-4\mathrm{i}} = \dfrac{3+4\mathrm{i}}{3-4\mathrm{i}} = \dfrac{(3+4\mathrm{i})^2}{(3-4\mathrm{i})(3+4\mathrm{i})} = \dfrac{-7+24\mathrm{i}}{25} = -\dfrac{7}{25} + \dfrac{24}{25}\mathrm{i}.$

3 Given the complex number: $z = \dfrac{m^2 - m - 6}{m+3} + (m^2 - 2m - 15)\mathrm{i}$:

(1) When z is a real number, find the value of real number m.

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(2) When z is a complex number, find the value of real number m.

(3) When z is a imaginary number, find the value of real number m.

Answer (1) When $m=5$, z is a real number.

(2) When $m\neq 5$ and $m\neq -3$, z is an complex number.

(3) When $m=3$ or $m=-2$, z is an imaginary number.

Solution (1) $\begin{cases} m+3\neq 0 \\ m^2-2m-15=0 \end{cases} \Rightarrow m=5.$

(2) $\begin{cases} m+3\neq 0 \\ m^2-2m-15\neq 0 \end{cases} \Rightarrow m\neq 5$ and $m\neq -3.$

(3) $\begin{cases} \frac{m^2-m-6}{m+3}=0 \\ m^2-2m-15\neq 0 \end{cases} \Rightarrow m=3$ or $m=-2.$

4 On the complex plane, point A represents the complex number:

$z=(x^2-2x-3)+(x^2+3x+2)\mathrm{i}.$

(1) When z is a real number, find the value of real number x.

(2) If point A is belong to Quadrant II, find the range of x.

Answer (1) $x=-2$ or -1.

(2) $x\in(-1,3)$.

Solution (1) $x^2+3x+2=0$, $x=-2$ or -1.

(2) $\begin{cases}x^2-2x-3<0\\x^2+3x+2>0\end{cases}$, so $-1<x<3$.

5 On the complex plane, which Quadrant the point represents the complex number $\frac{\mathrm{i}}{1-\mathrm{i}}$ belongs to?

A. Quadrant I B. Quadrant II C. Quadrant III D. Quadrant IV

Answer B

Solution $\frac{\mathrm{i}}{1-\mathrm{i}}=\frac{\mathrm{i}(1+\mathrm{i})}{(1-\mathrm{i})(1+\mathrm{i})}=\frac{-1+\mathrm{i}}{2}=-\frac{1}{2}+\frac{\mathrm{i}}{2}$.

For complex number: $\frac{\mathrm{i}}{1-\mathrm{i}}$, the real part is $-\frac{1}{2}$, the imaginary part os $\frac{1}{2}$.

so the point represents the complex number: $\frac{\mathrm{i}}{1-\mathrm{i}}$ belongs to Quadrant II.

Preview

6 Given that complex number z satisfies $|z-1|=1$, what is the maximum value of $|2z-2-4\mathrm{i}|$?

A. 5　　B. 6　　C. 7　　D. 8

Answer B

Solution Let's assume $z=a+bi$, then from $|z-1|=1$ we have:

$|(a-1)+bi|^2=1$, which gives $|(a-1)^2+b^2|=1$ (where $-1\leqslant b\leqslant 1$),

$|2z-2-4i|=2|z-1-2i|=2\sqrt{(a-1)^2+(b-2)^2}=2\sqrt{1-b^2+b^2-4b+4}=2\sqrt{5-4b}$.

Since $-1\leqslant b\leqslant 1$, we have $1\leqslant 5-4b\leqslant 9$,

Therefore, $2\sqrt{5-4b}\leqslant 2\cdot 3=6$.

Thus, the maximum value of $|2z-2-4i|$ is 6,

Therefore, the answer is B.

7 Assume m and $n\in\mathbf{R}$, two roots of the equation $x^2+mx+n=0$(respects with x) are α and β, respectively.

(1) When $\alpha=1+\mathrm{i}$, find the value of β, m , $and\ n$.

(2) When $m=2$, $n=4$, find the value of $|\alpha|+|\beta|$.

Answer (1) $\beta=1-\mathrm{i}$, $m=-2$, $n=2$.

(2) 4 .

Solution (1) We know that $\alpha=1+\mathrm{i}$ is a root of the equation $x^2+mx+n=0$,

Therefore, $(1+\mathrm{i})^2+m(1+\mathrm{i})+n=0$,

This simplifies to $m+n+(m+2)\mathrm{i}=0$,

Rearranging the equation, we have $m+n+(m+2)\mathrm{i}=0$,

Since m and n are real numbers,

We have $\begin{cases} m+n=0 \\ m+2=0 \end{cases}$,

Which gives $\begin{cases} m=-2 \\ n=2 \end{cases}$,

Thus, the equation in terms of x is $x^2-2x+2=0$,

Using the relationship between the roots and coefficients, we have $\alpha+\beta=2$,

Therefore, $\beta=2-\alpha=2-(1+\mathrm{i})=1-\mathrm{i}$,

In conclusion, $\beta=1-\mathrm{i}$, $m=-2$, $n=2$.

(2) When $m=2$ and $n=4$, the equation becomes $x^2+2x+4=0$,

The roots of the equation $x^2+2x+4=0$ are $x=\dfrac{-2\pm\sqrt{-2^2+4\times 4}\mathrm{i}}{2}$,

Simplifying, we have $x=-1\pm\sqrt{3}\mathrm{i}$,

Let's assume $\alpha=-1-\sqrt{3}\mathrm{i}$ and $\beta=-1+\sqrt{3}\mathrm{i}$,

Then, $|\alpha|+|\beta|=|-1-\sqrt{3}\mathrm{i}|+|-1+\sqrt{3}\mathrm{i}|$

$=\sqrt{(-1)^2+(-\sqrt{3})^2}+\sqrt{(-1)^2+(\sqrt{3})^2}=4$,

In conclusion, the value of $|\alpha|+|\beta|$ is 4.

Think Academy

Day 5

Review

1 Solve the quadratic equations.

(1) $3x^2+2x-5=0$

(2) $x(x-1)=2-2x$

Answer (1) $x_1=1$, $x_2=-\frac{5}{3}$.

(2) $x_1=1$, $x_2=-2$.

Solution (1) $3x^2+2x-5=0$,

$(x-1)(3x+5)=0$,

$\therefore x_1=1$, $x_2=-\frac{5}{3}$.

(2) $x^2-x=2-2x$,

$x^2-x+2x-2=0$,

$x^2+x-2=0$,

$(x+2)(x-1)=0$,

$\therefore x_1=1$, $x_2=-2$.

2 If the equation $(m-2)x^2-2x+1=0$ has two distinct real roots, what is the range of m?

A. $m < 3$ B. $m < 3$ and $m \neq 2$ C. $m \leqslant 3$ D. $m \leqslant 3$ and $m \neq 2$

Answer B

Solution According to the given conditions, we have $m-2 \neq 0$ and $\Delta = (-2)^2 - 4(m-2) > 0$,

Solving these inequalities, we obtain $m < 3$ and $m \neq 2$.

3 Given the function: $f(x) = (m^2 + 2m) \cdot x^{m^2+m-1}$:

(1) If $f(x)$ is a proportional function, what is the value of real number m?

(2) If $f(x)$ is a inverse proportional function, what is the value of real number m?

(3) If $f(x)$ is a quadratic function, what is the value of real number m?

Answer (1) $m = 1$.

(2) $m = -1$.

Think Academy

(3) $m=\frac{-1\pm\sqrt{13}}{2}$.

Solution (1) $\begin{cases} m^2+m-1=1 \\ m^2+2m\neq 0 \end{cases}$, so $m=1$.

(2) $\begin{cases} m^2+m-1=-1 \\ m^2+2m\neq 0 \end{cases}$, $\therefore m=-1$.

(3) $\begin{cases} m^2+m-1=2 \\ m^2+2m\neq 0 \end{cases}$, $\therefore m=\frac{-1\pm\sqrt{13}}{2}$.

4 What are the maximum and minimum values of the function $y=-x^2+4x+5$, where $x\in[1,4]$, respectively?

A. 8 , 9 B. 5 , 9 C. 5 , 8 D. 1 , 8

Answer B

Solution Since the graph of the quadratic function $y=-x^2+4x+5$ is a downward-opening parabola with the axis of symmetry at $x=2$, and considering that $x\in[1,4]$, we can conclude that the function has a maximum value of 9 when $x=2$, and a minimum value of 5 when $x=4$.

Therefore, the answer is B.

5 Given the function $f(x)=x^2-2kx+2$, it is known that for $x\geqslant -1$, and $f(x)\geqslant k$. Find the range of real numbers for k.

Answer $[-3,1]$.

Solution Let $g(x)=f(x)-k=x^2-2kx+2-k$,

then $g(x) \geqslant 0$ holds for all $x \geqslant -1$,

The graph of $g(x)$ has a symmetry axis at $x = k$,

Therefore, we have $\begin{cases} k \leqslant -1 \\ g(-1) \geqslant 0 \end{cases}$ or $\begin{cases} k > -1 \\ \Delta = 4k^2 - 4(2-k) \leqslant 0' \end{cases}$

Solving the inequalities, we obtain $-3 \leqslant k \leqslant 1$.

Preview

6 Given the inequality:$ax^2 + 3x + 2 > 0 (a \in \mathbf{R})$, with respect to x.

(1) The solution set of the inequality $ax^2 + 3x + 2 > 0$ is given by $\{x | b < x < 1\}$. Find the values of a and b.

(2) The solution set of the inequality $ax^2 + 3x + 2 > -ax - 1$ (where $a > 0$) is as follows.

Answer (1) $b = -\frac{2}{5}$.

(2) ① When $0 < a < 3$, the solution set is $\left\{x | x < -\frac{3}{a} \text{ or } x > -1\right\}$.

② When $a = 3$, the solution set is $\{x | x \neq -1\}$.

③ When $a > 3$, the solution set is $\left\{x | x < -1 \text{ or } x > -\frac{3}{a}\right\}$.

Solution (1) Substituting $x = 1$ into $ax^2 + 3x + 2 = 0$, we obtain $a = -5$.

Therefore, the inequality $ax^2 + 3x + 2 > 0$ becomes $-5x^2 + 3x + 2 > 0$.

Further simplifying, we have $(x-1)(5x+2)<0$.

Hence, the solution set of the original inequality is $\left\{x\middle|-\frac{2}{5}<x<1\right\}$.

Therefore, $b=-\frac{2}{5}$.

(2) The inequality $ax^2+3x+2>-ax-1$ can be rewritten as $ax^2+(a+3)x+3>0$

,

which can be further simplified to $(ax+3)(x+1)>0$.

When $0<a<3$, we have $-\frac{3}{a}<-1$, and the solution set of the inequality is $\left\{x|x>-1 \text{ or } x<-\frac{3}{a}\right\}$.

When $a=3$, we have $-\frac{3}{a}=-1$, and the solution set of the inequality is $\{x|x\neq-1\}$.

When $a>3$, we have $-\frac{3}{a}>-1$, and the solution set of the inequality is $\left\{x|x<-1 \text{ or } x>-\frac{3}{a}\right\}$.

In summary, the solution sets of the original inequality are:

① When $0<a<3$, $\left\{x|x<-\frac{3}{a} \text{ or } x>-1\right\}$.

② When $a=3$, $\{x|x\neq-1\}$.

③ When $a>3$, $\left\{x|x<-1 \text{ or } x>-\frac{3}{a}\right\}$.

7 Given the function: $f(x)=\begin{cases}-x^2+2x, x\leqslant a\\ x, x>a\end{cases}$.

(1) When $a=1$, the range of $f(x)$ is ______ .

(2) If the graph of the function $f(x)$ has only one common point with the line $y=a$, the range of the real number a is ______ (to be determined).

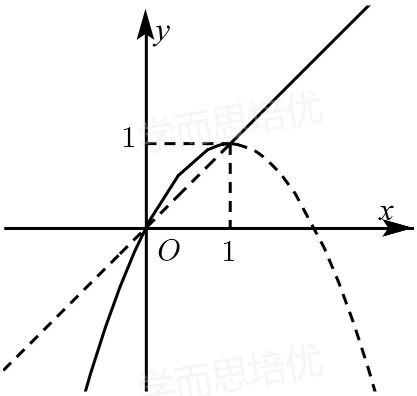
Answer (1) $\mathbf{R}$

(2) $[0, 1]$

Solution (1) When $a = 1$, the function $f(x) = \begin{cases} -x^2 + 2x, x \leqslant 1 \\ x, x > 1 \end{cases}$.

The graph of $y = f(x)$ is shown in the figure.

From the graph, we can see that the range of the function $f(x)$ is $\mathbf{R}$ (all real numbers).



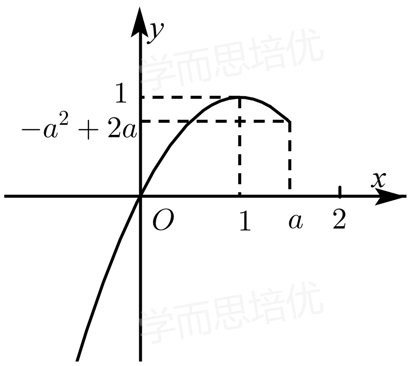
(2) Given that when $x > a$, $f(x) = x > a$, the graph of $f(x)$ does not intersect the line $y = a$.

Hence, if the graph of the function $f(x)$ has only one common point with the line $y = a$, it must satisfy $f(x) = -x^2 + 2x$ for $x \leqslant a$.

The symmetry axis of $f(x) = -x^2 + 2x$ is $x = 1$.

When $a \leqslant 1$, $f(x) \in (-\infty, -a^2 + 2a]$. For $f(x) = -x^2 + 2x$ to have only one common point with the line $y = a$, it must satisfy $a \leqslant -a^2 + 2a$, which implies $0 \leqslant a \leqslant 1$.

When $a > 1$, the graph of $f(x) = -x^2 + 2x$ is as shown in the graph.



In this case, for $f(x)=-x^2+2x$ to have only one common point with the line $y=a$, it must satisfy $a \leqslant -a^2+2a$. However, there is no solution for this condition.

In conclusion, the range of the real number a is $[0,1]$.

Think Academy

Day 6

Review

1 Given seven cards labeled with numbers -3, -2, -1, 0, 1, 2, 3, all having the same backside, the probability of drawing a card with an absolute value less than 2 is ______ .

Answer $\frac{3}{7}$

Solution The cards with an absolute value less than 2 are -1, 0, 1. Therefore, the probability is $\frac{3}{7}$.

2 In an opaque bag, there are 3 red balls and 1 yellow ball. They only differ in color. Two balls are randomly drawn from the bag. What is the probability of drawing exactly one yellow ball and one red ball?

A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{6}$

Answer A

Solution When randomly drawing two balls, there are a total of 6 possibilities. Out of these, there are exactly 3 possibilities where we draw one yellow ball and one red ball.

Therefore, the probability is $\frac{3}{6}=\frac{1}{2}$.

In other words, there is a 50% chance of drawing one yellow ball and one red ball.

3 The probability of selecting student A from a group of 5 students, including A and B, at random to choose 2 people is ( ) .

A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{8}{25}$ D. $\frac{9}{25}$

Answer B

Solution The total number of possible outcomes is $n=\binom{5}{2}=10$.

The number of outcomes where student A is selected is $m=\binom{1}{1}\binom{4}{1}=4$.

Therefore, the probability of selecting student A is $P=\frac{m}{n}=\frac{4}{10}=\frac{2}{5}$.

Hence, the answer is B.

4 In a box, there are 10 ping pong balls, including 6 new balls and 4 old balls. Two balls are randomly drawn without replacement for use. Given that a new ball is drawn on the first draw, what is the probability of drawing another new ball on the second draw?

A. $\frac{3}{5}$ B. $\frac{1}{10}$ C. $\frac{5}{9}$ D. $\frac{2}{5}$

Answer C

Solution Given that a new ball is drawn on the first draw, there are 9 balls remaining in the box, consisting of 5 new balls and 4 old balls.

Therefore, the probability of drawing another new ball on the second draw is $\frac{5}{9}$.

Hence, the answer is option C.

5 When tossing a fair coin three times in succession, what is the probability of getting at least one head?

A. $\frac{1}{8}$ B. $\frac{3}{8}$ C. $\frac{5}{8}$ D. $\frac{7}{8}$

Answer D

Solution The complementary event to "getting all tails in three tosses" is "getting at least one head." The probability of getting all tails is $\overline{P}=\left(\frac{1}{2}\right)^3=\frac{1}{8}$. Therefore, the probability of getting at least one head is $P=1-\overline{P}=1-\frac{1}{8}=\frac{7}{8}$.

Hence, the answer is D.

Preview

6 Put 7 same balls into 4 different boxes, marked as A, B, C, and D, respectively. Each box has 1 ball, at least. What is the probability of there are 3 balls in box A?

A. $\frac{3}{10}$ B. $\frac{2}{5}$ C. $\frac{3}{20}$ D. $\frac{1}{4}$

Answer C

Solution The number of ways to distribute 7 identical balls into 4 distinct boxes, with at least 1 ball in each box, is $\mathrm{C}_6^3=20$. Among these ways, the number of ways to have exactly 3 balls in box A is $\mathrm{C}_3^2=3$.

Therefore, the probability is $\frac{3}{20}$.

Thus, the answer is C.

7 In an opaque cloth bag, there are a total of 200 balls, consisting of red, yellow, and blue balls. Except for their colors, all the balls are identical. After conducting multiple experiments of drawing balls, a student observed that the frequency of drawing red balls stabilized at 35% and the frequency of drawing blue balls stabilized at 55%. Therefore, the number of possible yellow balls in the bag is ______ .

Answer 20

Solution Since the student observed that the frequency of drawing red balls stabilized at 35% and the frequency of drawing blue balls stabilized at 55%,

Therefore, the probability of drawing a yellow ball is $1 - 35\% - 55\% = 10\%$,

Therefore, the number of yellow balls in the bag is $200 \times 10\% = 20$,

So, the possible number of yellow balls in the bag is 20.

Hence, the answer is 20.

Day 7

Review

1 ${}_5C_5 +_6 C_5 +_7 C_5 +_8 C_5 = ($　$)$.

A. 28　　B. 126　　C. 84　　D. 70

Answer C

Solution ${}_5C_5 +_6 C_5 +_7 C_5 +_8 C_5 =_6 C_6 +_6 C_5 +_7 C_5 +_8 C_5 =_7 C_6 +_7 C_5 +_8 C_5 =_8 C_6 +_8 C_5 =_9 C_3$
$= 84$

2 Selecting 2 students randomly from a group of 5 students, A, B, C, D, and E. What is the probability of selecting A?

A. $\frac{1}{5}$　　B. $\frac{2}{5}$　　C. $\frac{8}{25}$　　D. $\frac{9}{25}$

Answer B

Solution Selecting 2 students randomly from a group of 5 students: A, B, C, D, and E.

The total number of possible outcomes is $n =_5 \mathrm{C}_2 = 10$.

The number of outcomes where A is selected is $m =_4 \mathrm{C}_1 = 4$.

Therefore, the probability of selecting A is $p = \frac{m}{n} = \frac{4}{10} = \frac{2}{5}$.

3 Companies A, B, and C are bidding for 9 projects. Company A bids for 3 projects, Company B bids for 2 projects, and Company C bids for 4 projects. How many different bidding combinations are there in total?

Answer $1260.$

Solution ${}_9C_3 \cdot {}_6C_2 \cdot {}_4C_4 = 1260.$

4 The four roommates each write a greeting card. The cards are then collected and redistributed so that each person receives a card from someone else. The number of different ways to distribute the four greeting cards is ______ .

Answer 9

Solution This problem can be seen as filling the numbers 1, 2, 3, 4 into the four squares labeled 1, 2, 3, 4. Each square should be filled with a number, and the filling should follow the rule that the number in each square is different from its label.

Therefore, in the first step, there are 3 ways to fill the number 1 into the 3 squares labeled 2 to 4.

In the second step, we fill the corresponding number (2, 3, or 4) into the remaining 3 squares, and there are 3 ways to do so.

In the third step, we fill the remaining 2 numbers into the remaining 2 squares, and there is only one way to do this.

Therefore, there are a total of 9 ways.

Think Academy

5 There are 6 numbers: 0, 1, 2, 3, 4, 5.

(1) How many odd three-digit numbers can be formed without repeating any digits?

(2) How many natural numbers less than 1000 can be formed with non-repeating digits?

Answer (1) 48

(2) 131.

Solution (1) Dividing it into three steps:

① First, choose the units digit, which can be done in 3 ways.

② Then, choose the hundreds digit, which can be done in 4 ways.

③ Finally, choose the tens digit, also in 4 ways.

Therefore, there are a total of 3 x 4 x 4 = 48 odd three-digit numbers.

(2) There are three categories:

① Single-digit numbers, with a total of 6 numbers;

② Two-digit numbers, with a total of $5\times5=25$ numbers;

③ Three-digit numbers, with a total of $5\times5\times4=100$ numbers.

Therefore, the total number of natural numbers less than 1000 that can be formed is $6+25+100=131$.

Preview

6 There are ______ different ways to arrange 6 students in a row, where student A is adjacent to student B and student A is not adjacent to student C.

Answer 192

Solution Using the bundling method, there are $A_5^5A_2^2 = 240$ ways when student A is adjacent to student B. When student A is adjacent to both student B and student C, there are $2A_4^4 = 48$ ways. Therefore, the total number of different arrangements is

$240 - 48 = 192$.

Hence, the answer is 192.

7 There are a total of 16 different cards, including 4 red, 4 yellow, 4 blue, and 4 green cards. We want to select 3 cards, satisfying the condition that the 3 cards cannot be of the same color, and there can be at most 1 red card. How many different ways to select the cards are there?

A. 232 B. 252 C. 472 D. 484

Answer C

Solution If there are no red cards, we need to select 3 cards from the yellow, blue, and green cards. If all three cards are of different colors, there are ${}_4C_1 \times_4 C1 \times_4 C1 = 64$ possibilities. If two cards are of the same color, there are ${}_3C_2 \times_2 C_1 \times_4 C_2 \times_4 C_1 = 144$ possibilities.

If there is one red card and the remaining two cards are of different colors, there are $_4\mathrm{C}_1 \times_3 \mathrm{C}_2 \times_4 \mathrm{C}_1 \times_4 \mathrm{C}_1 = 192$ possibilities. If the remaining two cards are of the same color, there are $_4\mathrm{C}_1 \times_3 \mathrm{C}_1 \times_4 \mathrm{C}_2 = 72$ possibilities.

Therefore, there are a total of $64 + 144 + 192 + 72 = 472$ different ways to select the cards.

By considering the cases without red cards and with one red card separately, we have $_{12}\mathrm{C}_3 - 3 \times_4 \mathrm{C}_3 +_4 \mathrm{C}_1 \times_{12} \mathrm{C}_2 = 472$.