# 2023 AMC 8 Solution

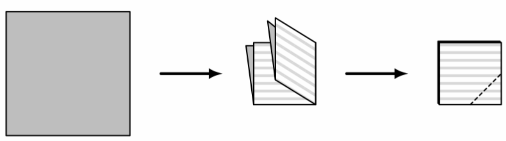
1. What is the value of $(8 \times 4 + 2) - (8 + 4 \times 2)$ ?

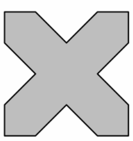
**(A)** 0 **(B)** 6 **(C)** 10 **(D)** 18 **(E)** 24

D

$$(8\times4+2)-(8+4\times2)$$
$$=(32+2)-(8+8)$$
$$=34-16$$
$$=18$$

2. A square piece of paper is folded twice into four equal quarters, as shown below, then cut along the dashed line. When unfolded, the paper will match which of the following figures?



(A)

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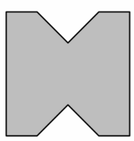
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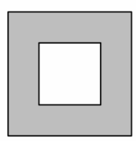

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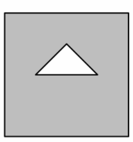
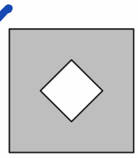
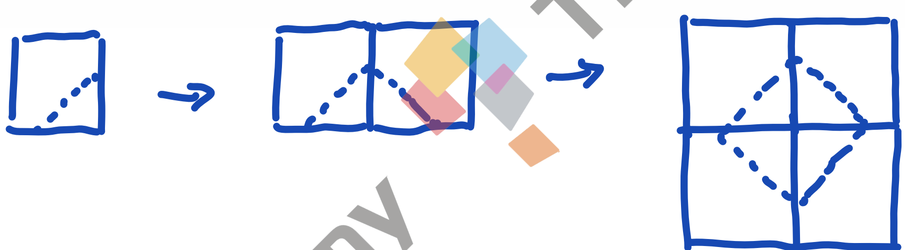

(E)




3. *Wind chill* is a measure of how cold people feel when exposed to wind outside. A good estimate for wind chill can be found using this calculation:

$$\text{(wind chill)} = \text{(air temperature)} - 0.7 \times \text{(wind speed)},$$

where temperature is measured in degrees Fahrenheit (°F) and wind speed is measured in miles per hour (mph). Suppose the air temperature is 36°F and the wind speed is 18 mph. Which of the following is closest to the approximate wind chill?

**(A)** 18 **(B)** 23 **(C)** 28 **(D)** 32 **(E)** 35

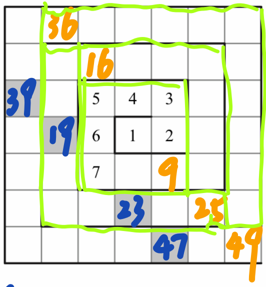
$36 - 0.7 \times 18$

$= 36 - 12.6$

$= 23.4$

$\approx 23$

4. The numbers from 1 to 49 are arranged in a spiral pattern on a square grid, beginning at the center. The first few numbers have been entered into the grid below. Consider the four numbers that will appear in the shaded squares, on the same diagonal as the number 7. How many of these four numbers are prime?




**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

Find the key:

Perfect squares

Then, we can find the four numbers: 19, 23, 39, 47.

Among them, 19, 23, 47 are prime.

5. A lake contains 250 trout, along with a variety of other fish. When a marine biologist catches and releases a sample of 180 fish from the lake, 30 are identified as trout. Assume that the ratio of trout to the total number of fish is the same in both the sample and the lake. How many fish are there in the lake?

**(A)** 1250 **(B)** 1500 **(C)** 1750 **(D)** 1800 **(E)** 2000

Trout : Total = 30 : 180 = 1 : 6

Trout : 250

Total : $250 \div 1 \times 6 = 1500$

6. The digits 2, 0, 2, and 3 are placed in the expression below, one digit per box. What is the maximum possible value of the expression?

$$\square^{\square} \times \square^{\square}$$

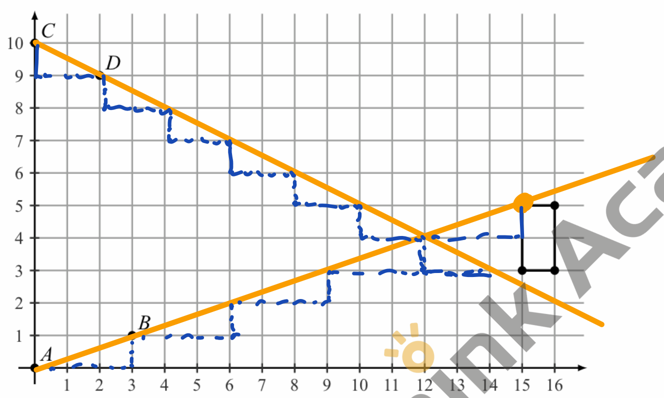
**(A)** 0 **(B)** 8 **(C)** 9 **(D)** 16 **(E)** 18

0 is special. To reach the maximum, 0 cannot be the base. Then, 0 should be the exponent. $\square^0 = 1$.

Consider the other term: $3^2 > 2^3 > 2^2$

Thus, the maximum is $3^2 \times 2^0 = 9$.

7. A rectangle, with sides parallel to the $x$-axis and $y$-axis, has opposite vertices located at $(15, 3)$ and $(16, 5)$. A line is drawn through points $A(0, 0)$ and $B(3, 1)$. Another line is drawn through points $C(0, 10)$ and $D(2, 9)$. How many points on the rectangle lie on at least one of the two lines?




**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

As shown above, connect A and B, C and D, and extend them. When extending, mind the proportional relationship between the grids (shown by the dotted line).

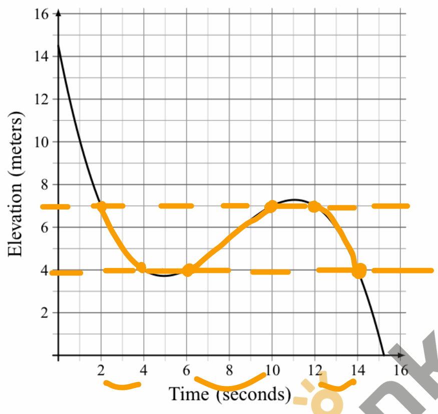
8. Lola, Lolo, Tiya, and Tiyo participated in a ping pong tournament. Each player competed against each of the other three players exactly twice. Shown below are the win-loss records for the players. The numbers 1 and 0 represent a win or loss, respectively. For example, Lola won five matches and lost the fourth match. What was Tiyo's win-loss record?

| Player | Result |
|---|---|
| Lola | 111011 |
| Lolo | 101010 |
| Tiya | 010100 |
| Tiyo | ?????? |
| | 000101 |

(A) 000101 (B) 001001 (C) 010000 (D) 010101 (E) 011000

Key: For each column, there are two games being played. Each game must have 1 win and 1 loss. Thus, on each column, there must be 2 wins and 2 losses.

9. Malaika is skiing on a mountain. The graph below shows her elevation, in meters, above the base of the mountain as she skis along a trail. In total, how many seconds does she spend at an elevation between 4 and 7 meters?




(A) 6 (B) 8 (C) 10 (D) 12 (E) 14

As shown below, it is in $\begin{cases} 2\text{-}4 \text{ min} \\ 6\text{-}10 \text{ min} \\ 12\text{-}14 \text{ min} \end{cases}$

In total $(4-2)+(10-6)+(14-12) = 8$ sec.

10. Harold made a plum pie to take on a picnic. He was able to eat only $\frac{1}{4}$ of the pie, and he left the rest for his friends. A moose came by and ate $\frac{1}{3}$ of what Harold left behind. After that, a porcupine ate $\frac{1}{3}$ of what the moose left behind. How much of the original pie still remained after the porcupine left?

**(A)** $\frac{1}{12}$ **(B)** $\frac{1}{6}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{3}$ **(E)** $\frac{5}{12}$

$$\left(1-\frac{1}{4}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{3}\right)$$

$$=\frac{3}{4}\times\frac{2}{3}\times\frac{2}{3}$$

$$=\frac{1}{3}$$

11. NASA's Perseverance Rover was launched on July 30, 2020. After traveling 292,526,838 miles, it landed on Mars in Jezero Crater about 6.5 months later. Which of the following is closest to the Rover's average interplanetary speed in miles per hour?

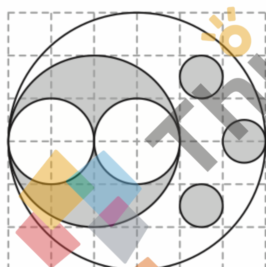
**(A)** 6,000 **(B)** 12,000 **(C)** 60,000 **(D)** 120,000 **(E)** 600,000

Distance: 292,526,838 ≈ 300,000,000

Time: 6.5 months ≈ 6.5 × 30 × 24 = 4,680 ≈ 5,000h

Speed: 300,000,000 ÷ 5,000 = 60,000 mph

12. The figure below shows a large white circle with a number of smaller white and shaded circles in its interior. What fraction of the interior of the large white circle is shaded?



**(A)** $\frac{1}{4}$ **(B)** $\frac{11}{36}$ **(C)** $\frac{1}{3}$ **(D)** $\frac{19}{36}$ **(E)** $\frac{5}{9}$

As shown below, suppose the red circle has an area of 1 unit.

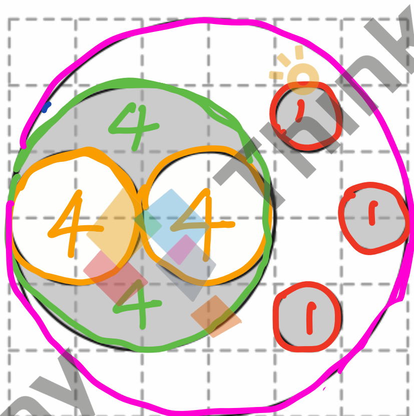
$$\begin{cases} r_{\text{yellow circle}} : r_{\text{red circle}} = 2:1 \\ S_{\text{yellow circle}} : S_{\text{red circle}} = 4:1 \end{cases} \Rightarrow$$ Each yellow circle's area is 4.

$$\begin{cases} r_{\text{green}} : r_{\text{red}} = 4:1 \\ S_{\text{green}} : S_{\text{red}} = 16:1 \end{cases} \Rightarrow S_{\text{green circle}} = 16$$

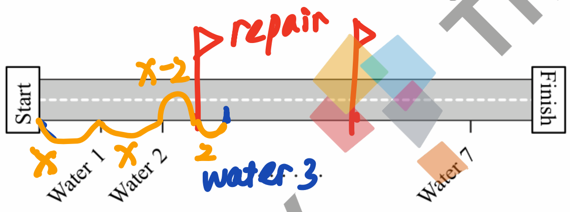
$$\begin{cases} r_{\text{pink}} : r_{\text{red}} = 6:1 \\ S_{\text{pink}} : S_{\text{red}} = 36:1 \end{cases} \Rightarrow S_{\text{pink circle}} = 36$$

The shaded area $= 1\times 3 + (16 - 2\times 4) = 11$

Fraction $= \dfrac{11}{36}$



13. Along the route of a bicycle race, 7 water stations are evenly spaced between the start and finish lines, as shown in the figure below. There are also 2 repair stations evenly spaced between the start and finish lines. The 3rd water station is located 2 miles after the 1st repair station. How long is the race in miles?




**(A)** 8 **(B)** 16 **(C)** 24 **(D)** 48 **(E)** 96

As shown above, the entire race can be represented as $(8x)$ or $3(2x+x-2)$.

An equation can be made:

$$8x = 3(2x+x-2)$$

$$x=6$$

The entire race is $8\times 6 = 48$ miles

14. Nicolas is planning to send a package to his friend Anton, who is a stamp collector. To pay for the postage, Nicolas would like to cover the package with a large number of stamps. Suppose he has a collection of 5-cent, 10-cent, and 25-cent stamps, with exactly 20 of each type. What is the greatest number of stamps Nicolas can use to make exactly \$7.10 in postage?

(Note: The amount \$7.10 corresponds to 7 dollars and 10 cents. One dollar is worth 100 cents.)

**(A)** 45 **(B)** 46 **(C)** 51 **(D)** 54 **(E)** 55

The total value of all collection is:

$$(5+10+25)\times 20 = 800 \text{ cents}$$

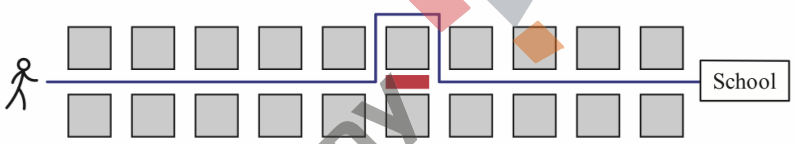
\$7.1 = 710 cents

$800-710 = 90$ cents, which means we need to take out 90 cents with as less stamps as possible:

$$90 = 25\times 3+10+5 \quad \boxed{\text{5 stamps needed}}$$

Thus, we need $20\times 3-5 = 55$ stamps at most.

15. Viswam walks half a mile to get to school each day. His route consists of 10 city blocks of equal length and he takes one minute to walk each block. Today, after walking 5 blocks, Viswam discovers that he has to make a detour, walking 3 blocks of equal length instead of 1 block to reach the next corner. From the time he starts his detour, at what speed, in miles per hour, must Viswam walk in order to arrive at school at his usual time?




**(A)** 4 **(B)** 4.2 ✓ **(C)** 4.5 **(D)** 4.8 **(E)** 5

1 block : $0.5 \div 10 = 0.05$ miles

From the time he starts detour, he needs to cover 7 blocks.

Distance $= 7 \times 0.05 = 0.35$ miles

Time $= 10 - 5 = 5$ min $= \frac{1}{12}$ hour

Speed $= 0.35 \div \frac{1}{12} = 4.2$ mph

16. The letters P, Q, and R are entered into a 20 × 20 table according to the pattern shown below. How many Ps, Qs, and Rs will appear in the completed table?

key: Each row is a repetition of a group 'PQR' with different orders.

| | | | | | |
|---|---|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ | ⋰ |
| Q | R | P | Q | R | ⋯ |
| P | Q | R | P | Q | ⋯ |
| R | P | Q | R | P | ⋯ |
| Q | R | P | Q | R | ⋯ |
| P | Q | R | P | Q | ⋯ |

Same

→ 6 group with RP left

→ 6 group with QR left

→ 6 group with PQ left

**(A)** 132 Ps, 134 Qs, 134 Rs

**(B)** 133 Ps, 133 Qs, 134 Rs

**(C)** 133 Ps, 134 Qs, 133 Rs ✓

**(D)** 134 Ps, 132 Qs, 134 Rs

**(E)** 134 Ps, 133 Qs, 133 Rs

In every three rows, P, Q, R each appears $6 \times 3 + 2 = 20$ times.

The table repeats by every three rows.

$20 \div 3 = 6R2$

The "three rows" repeats for 6 times with 2 rows left in 20x20 table.

For the 6 times repetition, P, Q, R each appears

$20 \times 6 = 120$ times.

For the 2 rows left, they are the same as the first 2 rows:

$$\begin{cases} 6 \text{ group with } QR \text{ left} \\ 6 \text{ group with } PQ \text{ left} \end{cases}$$

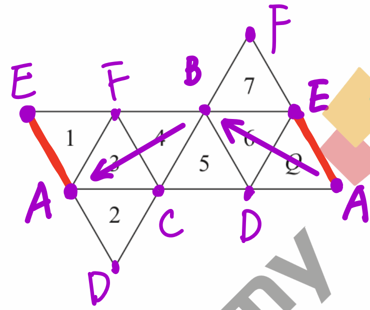
P appears: $6 \times 2 + 1 = 13$ times

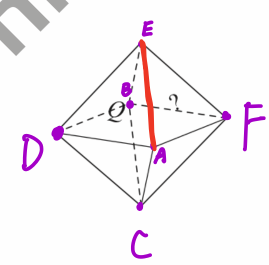
Q appears: $6 \times 2 + 2 = 14$ times

R appears: $6 \times 2 + 1 = 13$ times

In total
$$\begin{cases} P \text{ appears}: 120 + 13 = 133 \text{ times} \\ Q \text{ appears}: 120 + 14 = 134 \text{ times} \\ R \text{ appears}: 120 + 13 = 133 \text{ times.} \end{cases}$$

17. A *regular octahedron* has eight equilateral triangle faces with four faces meeting at each vertex. Jun will make the regular octahedron shown on the right by folding the piece of paper shown on the left. Which numbered face will end up to the right of $Q$?




✓ **(A)** 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

As shown above, the key is to find a face other than Q that shares line AE.

18. Greta Grasshopper sits on a long line of lily pads in a pond. From any lily pad, Greta can jump 5 pads to the right or 3 pads to the left. What is the fewest number of jumps Greta must make to reach the lily pad located 2023 pads to the right of her starting position?

**(A)** 405 **(B)** 407 **(C)** 409 **(D)** 411 **(E)** 413

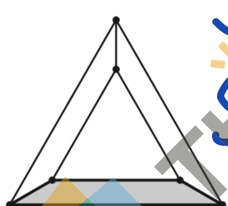
Suppose: to the right as "+5"
to the left as "-3"
The number of jumps to the right as $m$
The number of jumps to the left as $n$.
Then, $\boxed{5m - 3n = 2023}$

$\begin{cases} m = 407 \\ n = 4 \end{cases}$ $\begin{cases} m = 410 \\ n = 9 \end{cases}$ $\begin{cases} m = 413 \\ n = 14 \end{cases}$

total: 411 419 427 ....

Thus, the fewest number of jump is 411.

19. An equilateral triangle is placed inside a larger equilateral triangle so that the region between them can be divided into three congruent trapezoids, as shown below. The side length of the inner triangle is $\frac{2}{3}$ the side length of the larger triangle. What is the ratio of the area of one trapezoid to the area of the inner triangle?



$S_{inner\triangle} = \frac{4}{9} S_{big\triangle}$
$S_{\triangle} = (1 - \frac{4}{9}) \times \frac{1}{3} S_{big\triangle}$
$= \frac{5}{27} S_{big\triangle}$

**(A)** 1 : 3 **(B)** 3 : 8 **(C)** 5 : 12 **(D)** 7 : 16 **(E)** 4 : 9

Ratio: $\frac{5}{27} S_{big\triangle} : \frac{4}{9} S_{big\triangle}$
$= \frac{5}{27} : \frac{4}{9}$
$= 5 : 12$

20. Two integers are inserted into the list 3, 3, 8, 11, 28 to double its range. The mode and median remain unchanged. What is the maximum possible sum of the two additional numbers?

**(A)** 56 **(B)** 57 **(C)** 58 **(D)** 60 **(E)** 61

Mode: 3 Median: 8 Original Range: 28-3=25

Present Range: 25×2=50

Two new numbers(a+b) should be as large as possible.

Thus, the new order must be as shown below.

3 3 a 8 11 28 b

b = 50+3 = 53

$3 \le a < 8$. Thus, the maximum of a is 7.

a+b = 53+7 = 60

21. Alina writes the numbers 1, 2, . . ., 9 on separate cards, one number per card. She wishes to divide the cards into 3 groups of 3 cards so that the sum of the numbers in each group will be the same. In how many ways can this be done?

**(A)** 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

$(1+2+\cdots+9)\div 3 = 15$

For the group containing 1, the sum of the remaining 2 numbers must be 15−1=14, made of either ① 5&9 or ② 6&8.

① When we have 1, 5, 9, 2 must be with 6&7.

② When we have 1, 6, 8, 2 must be with 4&9.

Thus, we have 2 ways in total.

22. In a sequence of positive integers, each term after the second is the product of the previous two terms. The sixth term in the sequence is 4000. What is the first term?

**(A)** 1 **(B)** 2 **(C)** 4 **(D)** 5 **(E)** 10

We have a sequence:

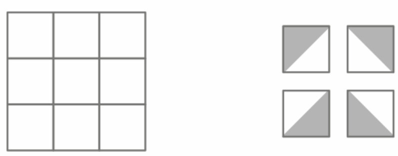
$a, \; b, \; ab, \; ab^2, \; a^2b^3, \; \boxed{a^3b^5}$

① ② ③ ④ ⑤ ⑥

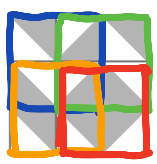
$a^3b^5 = 4000 = 2^5 \times 5^3$

$\begin{cases} a = 5 \\ b = 2 \end{cases}$

23. Each square in a $3 \times 3$ grid is randomly filled with one of the 4 gray-and-white tiles shown below on the right.



What is the probability that the tiling will contain a large gray diamond in one of the smaller $2 \times 2$ grids? Below is an example of such a tiling.



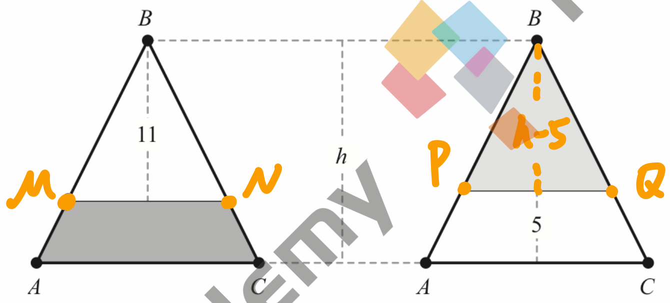
**(A)** $\frac{1}{1024}$ **(B)** $\frac{1}{256}$ **(C)** $\frac{1}{64}$ **(D)** $\frac{1}{16}$ **(E)** $\frac{1}{4}$

There are four large squares where diamond can appear (as shown above by four colors).

In each possible large square, the probability of getting a diamond is $(\frac{1}{4})^4 = \frac{1}{256}$.

The total probability is $\frac{1}{256} \times 4 = \frac{1}{64}$.

24. Isosceles triangle $ABC$ has equal side lengths $AB$ and $BC$. In the figures below, segments are drawn parallel to $\overline{AC}$ so that the shaded portions of $\triangle ABC$ have the same area. The heights of the two unshaded portions are 11 and 5 units, respectively. What is the height $h$ of $\triangle ABC$?




**(A)** 14.6 **(B)** 14.8 **(C)** 15 **(D)** 15.2 **(E)** 15.4

$\triangle BMN$ and $\triangle BAC$ are similar.

$\triangle BPQ$ and $\triangle BAC$ are similar.

$$S_{\triangle BMN} = \left(\frac{11}{h}\right)^2 S_{\triangle BAC} = \frac{121}{h^2} S_{\triangle BAC}$$

$$S_{\triangle BPQ} = \left(\frac{h-5}{h}\right)^2 S_{\triangle BAC} = \frac{(h-5)^2}{h^2} S_{\triangle BAC}$$

$$S_{\triangle BMN} + S_{MNAC} = S_{\triangle BAC}$$

$$S_{\triangle BPQ} = S_{MNAC}$$

$$S_{\triangle BMN} + S_{\triangle BPQ} = S_{\triangle BAC}$$

$$\frac{121}{h^2} + \frac{(h-5)^2}{h^2} = 1$$

$$121 + (h-5)^2 = h^2$$

$$121 + h^2 - 10h + 25 = h^2$$

$$10h = 146$$

$$h = 14.6$$

25. Fifteen integers $a_1, a_2, a_3, \ldots, a_{15}$ are arranged in order on a number line. The integers are equally spaced and have the property that

$$1 \le a_1 \le 10, \quad 13 \le a_2 \le 20, \quad \text{and} \quad 241 \le a_{15} \le 250.$$

What is the sum of the digits of $a_{14}$?

**(A)** 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

$$231 \le a_{15} - a_1 \le 249$$

Suppose the difference between each two adjacent terms is $d$.

$$231 \le 14d \le 249$$

$$16.5 \le d \le 17.78$$

$$d = 17$$

$$a_{15} = a_1 + 14 \times 17 = a_1 + 238$$

$$241 \le a_1 + 238 \le 250$$

$$3 \le a_1 \le 12$$

$$a_2 = a_1 + 17$$

$$13 \le a_1 + 17 \le 20$$

$$-4 \le a_1 \le 3$$

$a_1$ can only be 3.

$$a_{14} = a_1 + 13 \times 17 = 3 + 221 = 224$$

The digit sum is $2+2+4=8$.