# 2025 June AMC10 Daily Practice

## Week1 Day 1 - Number Operations

1 （1分）$\dfrac{53^2-47^2}{61^2-39^2}=$（　　）.

A. $\dfrac{3}{11}$　　B. $\dfrac{5}{11}$　　C. $\dfrac{6}{11}$　　D. $\dfrac{7}{11}$　　E. $\dfrac{9}{11}$

Answer A

Solution $\dfrac{53^2-47^2}{61^2-39^2}=\dfrac{(53-47)(53+47)}{(61-39)(61+39)}=\dfrac{6\times100}{22\times100}=\dfrac{6}{22}=\dfrac{3}{11}.$

2 （1分）Calculate:

$(2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+3\sqrt{2})+(\sqrt{2}+\sqrt{3})^2+\sqrt{(2-\sqrt{6})^2}=$_____.

A. $-4+3\sqrt{6}$　　B. $-3+3\sqrt{3}$　　C. $-3+3\sqrt{6}$　　D. $-2+3\sqrt{2}$　　E. $-3+4\sqrt{6}$

Answer C

Solution $\because (2\sqrt{3}-3\sqrt{2})(2\sqrt{3}+3\sqrt{2})+(\sqrt{2}+\sqrt{3})^2+\sqrt{(2-\sqrt{6})^2}$

$=(2\sqrt{3})^2-(3\sqrt{2})^2+2+3+2\sqrt{6}+|2-\sqrt{6}|$

$=12-18+5+2\sqrt{6}+\sqrt{6}-2$

$=-3+3\sqrt{6}$

3 （1分）Calculate:

$(\sqrt{2}+\sqrt{3}+\sqrt{5})(\sqrt{2}+\sqrt{3}-\sqrt{5})(\sqrt{2}-\sqrt{3}+\sqrt{5})(-\sqrt{2}+\sqrt{3}+\sqrt{5})=$_____.

A. 22　　B. 23　　C. 24　　D. 25　　E. 26

Answer C

Solution $=\left[(\sqrt{2}+\sqrt{3})^2-5\right]\left[5-(\sqrt{2}-\sqrt{3})^2\right]=2\sqrt{6}\times2\sqrt{6}=24.$

4 （1分）Given $a+b=\sqrt{\sqrt{1992}+\sqrt{1991}}$, $a-b=\sqrt{\sqrt{1992}-\sqrt{1991}}$, $ab=$_____ .

A. $\sqrt{1991}$　　B. $\frac{1}{2}\sqrt{1991}$　　C. $\sqrt{498}$　　D. $2\sqrt{498}$　　E. $1$

Answer B

Solution Since $(a+b)^2=\sqrt{1992}+\sqrt{1991}$,

$(a-b)^2=\sqrt{1992}-\sqrt{1991}$,

$4ab=(a+b)^2-(a-b)^2$,

$ab=\frac{1}{4}\left[(a+b)^2-(a-b)^2\right]$,

$=\frac{1}{4}\left[\left(\sqrt{1992}+\sqrt{1991}\right)-\left(\sqrt{1992}-\sqrt{1991}\right)\right]$,

$=\frac{1}{2}\sqrt{1991}$.

5 （1分）Simplify：$\sqrt{23-6\sqrt{10+4\sqrt{3-2\sqrt{2}}}}=$（　　）.

A. $-\sqrt{2}$　　B. $1-\sqrt{2}$　　C. $3-\sqrt{2}$　　D. $4-\sqrt{2}$　　E. $2$

Answer C

Solution $\sqrt{23-6\sqrt{10+4\sqrt{3-2\sqrt{2}}}}$

$=\sqrt{23-6\sqrt{10+4\sqrt{(\sqrt{2}-1)^2}}}$

$=\sqrt{23-6\sqrt{10+4(\sqrt{2}-1)}}$

$=\sqrt{23-6\sqrt{(\sqrt{2}+2)^2}}$

$=\sqrt{23-6(\sqrt{2}+2)}$

$=\sqrt{11-6\sqrt{2}}$

$=\sqrt{(3-\sqrt{2})^2}$

$=3-\sqrt{2}$ .

# Week1 Day 2 - Exponents & Absolute Values

1 （1分）$2[2(2^3)^4]^5 \div \{2[2(2^5)^4]^3\} = (\quad)$ .

A. $1$ B. $2$ C. $2^2$ D. $2^3$ E. $2^4$

Answer C

Solution $2[2(2^3)^4]^5 \div \{2[2(2^5)^4]^3\} = 2[2(2^{12})]^5 \div \{2[2(2^{20})]^3\} = 2^{66} \div 2^{64} = 2^2$.

2 （1分）Given $a$ is a nonzero real number, what is the value of $\frac{a}{|a|} + \frac{a^2}{|a^2|} + \frac{a^3}{|a^3|}$?

A. $3$ or $1$ B. $-3$ or $1$ C. $3$ or $-1$ D. $-3$ or $-1$ E. $3$

Answer C

Solution When $a > 0$, $|a| = a$,

$\therefore = 1 + 1 + 1 = 3$;

when $a < 0$, $|a| = -a$, $|a^3| = -a^3$,

$\therefore = -1 + 1 - 1 = -1$.

Choose $\mathbf{A}$.

3 （1分）Which expression is equal to $\sqrt{(x-1)^2} - \sqrt{(x-2)^2}$ for $|x-1| = 1-x$?

A. $3-2x$ B. $1$ C. $-1$ D. $2x-3$ E. $2x-1$

Answer C

Solution $\because |x-1| = 1-x$,

$\therefore x - 1 \leqslant 0$,

$x \leqslant 1$,

$\therefore x - 2 \leqslant -1$,

$\therefore \sqrt{(x-1)^2}-\sqrt{(x-2)^2}$

$=|x-1|-|x-2|$

$=1-x-2+x$

$=-1.$

Choose C.

4 （1分）$\left(\frac{7}{3}\right)^{1004}\sqrt{\frac{3^{2008}+15^{2008}}{7^{2008}+35^{2008}}}=$_____.

A. 1　　B. $\frac{7}{3}$　　C. $\frac{49}{9}$　　D. $\frac{3}{7}$　　E. $\frac{9}{49}$

Answer A

Solution Since $\frac{3^{2008}+15^{2008}}{7^{2008}+35^{2008}}=\frac{3^{2008}(1+5^{2008})}{7^{2008}(1+5^{2008})}=\frac{3^{2008}}{7^{2008}}$,

$=\left(\frac{7}{3}\right)^{1004}\cdot\left(\frac{3}{7}\right)^{1004}=1.$

5 （1分）Given $5^x=160, 32^y=160, (-2025)^{(x-1)(y-1)-1}=$_____.

A. 1　　B. 2025　　C. $\frac{1}{2025}$　　D. $-\frac{1}{2025}$　　E. $-2025$

Answer A

Solution $\because 5^x=160,$

$\therefore (5^x)^y=160^y,$

$\therefore 5^{xy}=(5\times 32)^y=5^y\times 32^y,$

$\because 5^x=160$ , $32^y=160,$

$\therefore 5^{xy}=5^y\times 160=5^y\times 5^x=5^{x+y},$

$\therefore xy=x+y,$

$\therefore (-2025)^{(x-1)(y-1)-1}$

$=(-2025)^{xy-x-y+1-1}$

$=(-2025)^{xy-(x+y)}$

$= (-2025)^0$

$= 1.$

## Week1 Day 3 - Equations and Inequalities

1 （1分）If $x$ is a real number and $|x^2 - 25| \leqslant 11$, what is the difference between the maximum value of $x$ and the minimum value of $x$?

A. $6$ B. $8$ C. $10$ D. $12$ E. $14$

Answer D

Solution $0 \leqslant x^2 - 25 \leqslant 11$,

$25 \leqslant x^2 \leqslant 36$,

$-6 \leqslant x \leqslant -5$ or $5 \leqslant x \leqslant 6$.

The difference between the maximum and minimum value of $x$ is $= 6 - (-6) = 12$.

2 （1分）$-k - 4x + 23 > 0$ is an inequality about $x$ and its positive integer solution is $x = 1$, $2$ or $3$. What is the sum of all possible values of positive integer $k$?

A. $23$ B. $27$ C. $34$ D. $37$ E. $42$

Answer C

Solution $-k - 4x + 23 > 0$ ,

$x < \frac{23 - k}{4}$ ,

$\because$ The positive integer solution of $x$ are $1$, $2$ and $3$,

then $3 < \frac{23 - k}{4} \leqslant 4$,

$\therefore$ the possible value of $k$ is $7 \leqslant k < 11$,

thus, the sum of all possible values of positive integer $k$ is $= 7 + 8 + 9 + 10 = 34$.

3 （1分）What is the sum of all possible solutions of $x$ for the equation:

$\sqrt{x^2+3x+7}-\sqrt{x^2-3x+4}=3$?

A. $-2$ B. $0$ C. $3$ D. $7$ E. $9$

Answer C

Solution Assume $\sqrt{x^2+3x+7}=a$, $\sqrt{x^2-3x+4}=b$,

$\therefore a^2-b^2=6x+3$,

$\therefore a+b=2x+1$,

$\therefore a=\sqrt{x^2+3x+7}=x+2$,

$\therefore x^2+3x+7=x^2+4x+4$,

$\therefore x=3$.

4 （1分）Given that the positive integers $x$ and $y$ satisfy the equation $4x^2-9y^2+20x+24y=8$, find the value of $x+y$.

A. $3$ B. $4$ C. $5$ D. $6$ E. $7$

Answer D

Solution $4x^2+20x+25-9y^2+24y-16=8+25-16$

$(2x+5)^2-(3y-4)^2=17$,

$[(2x+5)-(3y-4)]\,[(2x+5)+(3y-4)]=17$,

since $x$ and $y$ are positive integer, $\begin{cases}2x+5-3y+4=1\\2x+5+3y-4=17\end{cases}$,

$\begin{cases}x=2\\y=4\end{cases}$, $x+y=6$.

5 （1分）$a$ and $b$ are two real roots of the equation $2x^2+kx-2k+1=0$. And $a^2+b^2$ is $\frac{29}{4}$, then what is the sum of the possible value of $k$?

A. $-8$ B. $0$ C. $3$ D. $7$ E. $12$

Answer C

Solution ∵ the real roots of $2x^2+kx-2k+1=0$ are $a$, $b$,

$\therefore a+b=-\frac{k}{2}$, $ab=\frac{1-2k}{2}$,

$\therefore a^2+b^2=(a+b)^2-2ab=\frac{k^2}{4}-(1-2k)=\frac{1}{4}k^2+2k-1$,

$\because a^2+b^2=\frac{29}{4}$,

$\therefore \frac{1}{4}k^2+2k-1=\frac{29}{4}$,

$k^2+8k-4=29$,

$k^2+8k-33=0$,

$(k+11)(k-3)=0$,

$\therefore k=-11$ or $k=3$,

when $k=-11$, the function is $2x^2-11x+23=0$,

$\therefore \Delta=(-11)^2-4\times 2\times 23=121-184<0$,

no real roots, reject,

when $k=3$时, the function is $2x^2+3x-5=0$,

$\therefore \Delta=9+40=49>0$,

$\therefore k=3$ satisfies.

Thus, the sum of the possible value of $k$ is $3$.

## Week 2 Day 1 - Sequences

1 （1分）Lily wants to distribute $210$ chess pieces into several boxes. The first box contains $1$ piece, the second box contains $2$ pieces, the third box contains $3$ pieces, and so on. The pieces are exactly when placed in this manner. How many boxes does Lily use?

A. $18$ B. $19$ C. $20$ D. $21$ E. $22$

Answer C

Solution Assume Lily uses $n$ boxes.

The total number of pieces follows the arithmetic series: $1+2+3+\cdots+n=210$

Using the arithmetic series sum formula: $\frac{n(n+1)}{2}=210$

Multiply both sides by $2$: $n(n+1)=420$

Solve the quadratic equation: $n^2+n-420=0$

Factorizing: $(n-20)(n+21)=0$

Since $n>0$, we get $n=20$.

Final result: $\boxed{20}$

2 （1分）Given that both sequences $a_1, a_2, \ldots, a_{31}$ and $b_1, b_2, \ldots, b_{31}$ are arithmetic sequences, each containing $31$ terms. If $a_2+b_{30}=29$ and $a_{30}+b_2=-9$, find the total sum of these two sequences.

A. $29$ B. $60$ C. $200$ D. $310$ E. $400$

Answer D

Solution For arithmetic sequences, the sum of terms equidistant from the ends is constant:

$a_1+a_{31}=a_2+a_{30}, \quad b_1+b_{31}=b_2+b_{30}.$

By pairing terms symmetrically:

$(a_1+b_{31})+(b_1+a_{31})=(a_2+b_{30})+(b_2+a_{30})=29+(-9)=20.$

There are $15$ such pairs.

Including the middle terms (which also sum to $10$), the total sum is: $15\times 20+10=310.$

Final result: $\boxed{310}$

3 （1分）In a biology lab experiment, a petri dish initially contains $200$ bacteria. During daylight hours, the bacteria count decreases by $65$ due to UV exposure, while during nighttime hours, it increases by $40$ due to nutrient dispersion. On which daylight period will all bacteria perish?

A. $3$ B. $4$ C. $5$ D. $6$ E. $7$

Answer

E

Solution Let $a_n$ represent the bacteria count at the start of the $n$-th day.

The sequence follows this daily pattern:

Daylight Reduction: $a_n \to a_n - 65$

Nighttime Growth: If $a_n - 65 > 0$, then $a_{n+1} = (a_n - 65) + 40 = a_n - 25$.The process terminates when $a_n - 65 \leq 0$.

General Formula: The bacteria count forms an arithmetic sequence after the first day:

$a_n = 200 - 25(n-1)$ Solve for $n$ when $a_n - 65 \leq 0$:

$200 - 25(n-1) - 65 \leq 0 \implies 135 \leq 25(n-1) \implies n - 1 \geq 5.4 \implies n \geq 6.4$

Thus, extinction occurs on the $\lceil 6.4 \rceil = 7$-th day.

Final Answer: All bacteria perish during the $\boxed{7}$-th daylight period.

4 （1分）Alex has a $20$-week marathon training plan. The first week he runs $2$ kilometers. For the following weeks, he doubles his weekly distance until it exceeds $30$ kilometers. After that, he increases his weekly distance by $2$ kilometers each week until it exceeds the marathon distance ($42.195$ kilometers). Once exceeding the marathon distance, he maintains that weekly distance until the end of the training. What is the total distance Alex runs by the end of the training plan?

A. $724$ B. $736$ C. $660$ D. $682$ E. $692$

Answer E

Solution Phase 1:Geometric Sequence

First term ($a_1$): $2$ km , Common ratio ($r$): $2$ ,$a_n = 2 \times 2^{n-1} = 2^n$.

Determine $N_1$: Solve $2^{N_1} > 30 \implies N_1 = 5$.

Sum of Phase 1:$S_1 = a_1 \dfrac{r^{N_1} - 1}{r - 1} = 2 \times \dfrac{2^5 - 1}{2 - 1} = 62$ km.

Phase $2$: Arithmetic Sequence

First term ($a_1'$): $34$ km (Week 6, starting after Phase 1) , Common difference ($d$): $2$ km ,

General term: $a_k' = 34 + (k-1) \times 2$

Determine $N_2$: Solve $34 + 2(k-1) > 42.195 \implies k = 6$.

Thus, Phase 2 lasts $6$ weeks (Weeks 6 to 11).

Sum of Phase 2: $S_2 = 234$ km.

Phase $3$: Constant Sequence

Constant term ($C$): $44$ km, Number of weeks ($M$): $9$ weeks (Weeks 12 to 20)

Sum of Phase 3: $S_3 = C \times M = 44 \times 9 = 396$ km.

Total Distance: $S_{\text{total}} = S_1 + S_2 + S_3 = 62 + 234 + 396 = \boxed{692}$ kilometers.

5 （1分）A weather station records annual temperature changes (in degrees Celsius) relative to a baseline in a region. The first year's change is $+1^\circ\text{C}$, and the second year's change is $+2^\circ\text{C}$. Starting from the third year, the annual temperature change follows the rule: $\textbf{Year } (n+1) \textbf{ change} = \textbf{Year } n \textbf{ change} - \textbf{Year } (n-1) \textbf{ change} \quad (n \geq 2)$. What is the total cumulative temperature change after $2019$ years?

A. $3$ B. $4$ C. $5$ D. $6$ E. $7$

Answer B

Solution Let the temperature change sequence be $\{a_n\}$, defined as:

$a_1 = 1 \quad (\textbf{Change in Year 1}),$
$a_2 = 2 \quad (\textbf{Change in Year 2}),$
$a_{n+1} = a_n - a_{n-1} \quad (n \geq 2).$

Calculate Initial Terms to Identify Periodicity:

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $a_n$ | 1 | 2 | 1 | $-1$ | $-2$ | $-1$ | 1 | 2 |

The sequence has a period of $6$: $a_7 = a_1$, $a_8 = a_2$.

Each period of $6$ terms sums to: $S_6 = 1 + 2 + 1 + (-1) + (-2) + (-1) = 0.$

Since $2019 = 6 \times 336 + 3$, we have: $S_{2019} = 336 \times S_6 + (a_1 + a_2 + a_3)$
$= 336 \times 0 + (1 + 2 + 1)$
$= \boxed{4} \quad (\text{degrees Celsius}).$

## Week 2 Day 2 - Word Problems 1

1 （1分）A manufacturing plant must produce $2,220$ precision components. Three workers have different production rates, with their individual times to complete the task alone in the ratio $4:5:6$. If they collaborate simultaneously to finish the order, how many components does each worker produce by the time the job is completed?

A. $800, 720, 700$ B. $900, 820, 500$ C. $740, 740, 740$ D. $960, 660, 600$ E. $900, 720, 600$

Answer E

Solution Work Rate Ratios $\frac{1}{4}:\frac{1}{5}:\frac{1}{6}=15:12:10$

Since they work for the same duration, the number of components produced is proportional to their efficiencies:

$$\text{Total ratio units}=15+12+10=37$$

$$\frac{15}{37}\times 2220=900$$

$$\frac{12}{37}\times 2220=720$$

$$\frac{10}{37}\times 2220=600$$

Final Answer: $\boxed{900, 720, 600}$.

2 （1分）A delivery person needs to transport a package between two towns within a specified timeframe. Riding a motorcycle at $15km/h$ allows arrival $24$ minutes early, while riding at $12km/h$ causes a $15$-minute delay. What is the distance between the two towns?

A. $15$ B. $24$ C. $39$ D. $48$ E. $60$

Answer C

Solution Let $d$ represent the distance (in kilometers).

$$\frac{d}{12}-\frac{d}{15}=\left(\frac{15}{60}+\frac{24}{60}\right)\implies\frac{d}{60}=\frac{39}{60}\implies d=39$$

Final Result: The distance between the two towns is $\boxed{39}$ kilometers.

3 （1分）

Two alloys contain copper and zinc in the ratios $3:4$ and $5:3$ respectively. If equal weights of both alloys are melted together, what is the new ratio of copper to zinc in the combined alloy?

A. $59:53$ B. $56:53$ C. $49:53$ D. $56:43$ E. $49:43$

Answer A

Solution Let each alloy contribute $1$ unit of weight.

Calculate components in first alloy (3:4)

Copper: $\frac{3}{7}\times 1=\frac{3}{7}$ units, Zinc: $\frac{4}{7}\times 1=\frac{4}{7}$ units

Calculate components in second alloy (5:3)

Copper: $\frac{5}{8}\times 1=\frac{5}{8}$ units, Zinc: $\frac{3}{8}\times 1=\frac{3}{8}$ units

Sum total components:

Total copper: $\frac{3}{7}+\frac{5}{8}=\frac{24+35}{56}=\frac{59}{56}$

Total zinc: $\frac{4}{7}+\frac{3}{8}=\frac{32+21}{56}=\frac{53}{56}$

Simplify the ratio: $\frac{59}{56}:\frac{53}{56}=59:53$

Final Answer: The new ratio is $\boxed{59:53}$.

4 （1分）At a school club, if the number of male members decreases by one-third, the total membership becomes $22$. If the number of female members increases by one-third, the total membership becomes $32$. How many members are in the club?

A. $24$ B. $26$ C. $28$ D. $30$ E. $32$

Answer D

Solution Let $x$ represent the number of male members and $y$ represent the number of female members.

The problem translates to the system of equations: $\begin{cases}\frac{2}{3}x+y=22\\ x+\frac{4}{3}y=32\end{cases}$

1. Subtract the first equation from the second to eliminate fractions:

$\left(x+\frac{4}{3}y\right)-\left(\frac{2}{3}x+y\right)=32-22$

2. Simplify the left side: $x-\frac{2}{3}x+\frac{4}{3}y-y=\frac{1}{3}x+\frac{1}{3}y$

3. Combine terms and solve for $x+y$: $\frac{1}{3}(x+y)=10 \implies x+y=30$

Final Result: The total number of club members is $\boxed{30}$.

5 ( 1分 ) An electronics store offers a promotion: "For every $\$300$ spent, receive a $\$50$ gift card." Any amount over $\$300$ but less than the next $\$300$ is ignored. Gift cards can be used like cash for future purchases but cannot be combined with the promotion. A customer first buys a laptop for $\$1000$, receives gift cards, then uses those gift cards plus $\$280$ cash to buy a tablet. What discount percentage did the customer effectively get on both purchases?

A. $9\%$ B. $10.5\%$ C. $11.15\%$ D. $13.7\%$ E. $15\%$

Answer B

Solution The customer spent $\$1000$ on the laptop.

Number of $\$300$ increments: $\left\lfloor\frac{1000}{300}\right\rfloor=3$ Total gift cards received: $3\times\$50=\$150$.

Second purchase details: The customer uses $\$150$ in gift cards and $\$280$ cash to buy the tablet. Total spent on the tablet: $\$150+\$280=\$430$ .

Total value of purchases: Laptop value $=\$1000$, Tablet value$=\$430$, Total value $=\$1000+\$430=\$1430$.

Total cash paid by the customer:$\$1000+\$280=\$1280$.

Payment percentage $=\frac{1280}{1430}\times100\%\approx89.5\%$,

Discount percentage$=100\%-89.51\%\approx10.5\%$,

Final Answer: $\boxed{10.5\%}$.

## Week 2 Day 3 - Word Problems 2

1 ( 1分 ) Alex and Sam are discussing their ages. Alex states, "When I was your current age, you were only $4$ years old!" Sam responds, "And when I reach your current age, you will be $61$!"

What are their current ages?

A. $42, 23$ B. $42, 43$ C. $30, 60$ D. $21, 32$ E. $4, 61$

Answer A

Solution Let Alex's current age be $A$ and Sam's current age be $S$.

When Alex was Sam's current age ($S$), the time elapsed since then is $(A - S)$ years. At that time, Sam's age was $S - (A - S) = 2S - A = 4 \quad \textbf{(Equation 1)}$

When Sam reaches Alex's current age ($A$), the time required is $(A - S)$ years. At that time, Alex's age will be $A + (A - S) = 2A - S = 61 \quad \textbf{(Equation 2)}$

From Equation 1: $A = 2S - 4$.

Substitute $A = 2S - 4$ into Equation 2:

$2(2S - 4) - S = 61$
$4S - 8 - S = 61$
$3S = 69$
$S = 23$
$A = 2(23) - 4 = 42$

Final result: $\boxed{A = 42}$ and $\boxed{S = 23}$.

2 （1分）Harold baked a colossal blueberry pie for a barbecue. He first ate $\frac{1}{6}$ of it, then divided the remainder equally among his $4$ friends. Each friend ate $\frac{1}{3}$ of their portion. Later, a raccoon arrived and devoured $\frac{1}{5}$ of the remaining pie. Finally, a fox consumed $\frac{2}{9}$ of the raccoon's remaining pie. What fraction of the original pie remains intact?

A. $\frac{1}{5}$ B. $\frac{2}{9}$ C. $\frac{9}{28}$ D. $\frac{28}{81}$ E. $\frac{34}{81}$

Answer D

Solution Let $P$ represent the original quantity of the pie.

Harold eats $\frac{1}{6}$ of the pie, remaining pie: $P - \frac{1}{6}P = \frac{5}{6}P$

Divide the remainder among $4$ friends:

Each friend receives: $\frac{5}{6}P \div 4 = \frac{5}{24}P$, Each friend eats $\frac{1}{3}$ of their portion, leaving: $\frac{5}{24}P \times \left(1 - \frac{1}{3}\right) = \frac{5}{24}P \times \frac{2}{3} = \frac{10}{72}P = \frac{5}{36}P$, Total remaining after all friends eat: $4 \times \frac{5}{36}P = \frac{20}{36}P = \frac{5}{9}P$

Raccoon eats $\frac{1}{5}$ of the remaining pie: $\frac{5}{9}P \times \left(1 - \frac{1}{5}\right) = \frac{5}{9}P \times \frac{4}{5} = \frac{20}{45}P = \frac{4}{9}P$

Fox eats $\frac{2}{9}$ of the raccoon's remaining pie: $\frac{4}{9}P \times \left(1 - \frac{2}{9}\right) = \frac{4}{9}P \times \frac{7}{9} = \frac{28}{81}P$

Final Result: The fraction of the original pie that remains is $\boxed{\frac{28}{81}}$.

3 （1分）Three cups, Cup $A$, $B$, and $C$, contain $10$ grams, $20$ grams, and $30$ grams of water respectively. A $10$-gram solution of sugar water is poured into Cup $A$. After mixing thoroughly, $10$ grams of the mixture is transferred to Cup $B$. This process is repeated: $10$ grams from Cup $B$ is then transferred to Cup $C$. The final concentration of sugar in Cup $C$ is $0.5\%$. What was the initial concentration of the sugar water poured into Cup $A$?

A. $9\%$ B. $10\%$ C. $11\%$ D. $12\%$ E. $13\%$

Answer D

Solution Let $c$ be the initial concentration (in decimal) of the sugar water poured into Cup A.

Step 1: Pour into Cup A

Initial mixture in Cup A: Total mass = $10\text{g (water)} + 10\text{g (sugar water)} = 20\text{g}$. Sugar mass = $10c$ grams. Concentration in Cup A: $\frac{10c}{20} = \frac{c}{2}$

Step 2: Transfer to Cup B

10g taken from Cup A: Sugar transferred = $10 \times \frac{c}{2} = 5c$ grams. New mixture in Cup B: Total mass = $20\text{g (original water)} + 10\text{g (added)} = 30\text{g}$. Total sugar = $5c$ grams.

Concentration in Cup B: $\frac{5c}{30} = \frac{c}{6}$

Step 3: Transfer to Cup C

10g taken from Cup B: Sugar transferred = $10 \times \frac{c}{6} = \frac{5c}{3}$ grams. New mixture in Cup C: Total mass = $30\text{g (original water)} + 10\text{g (added)} = 40\text{g}$. Total sugar = $\frac{5c}{3}$ grams.

Concentration in Cup C: $\frac{\frac{5c}{3}}{40} = \frac{c}{24}$

Given the final concentration in Cup C is $0.5\% = 0.005$ in decimal: $\frac{c}{24} = 0.005$

Solve for $c$: $c = 0.005 \times 24 = 0.12 \quad (\text{i.e., } 12\%)$

Final Answer: The initial concentration of the sugar water was $\boxed{12\%}$.

4 （1分）A retail store sells a product at its original price, earning a profit of $25\%$ of the cost price. Later, the store reduces the price to $90\%$ of the original price. Despite this reduction, the total profit increases by $25\%$ compared to the original profit. What is the multiple of the new sales quantity relative to the original sales quantity?

A. $1$ B. $1.5$ C. $2$ D. $2.5$ E. $3$

Answer D

Solution Let $C$ be the cost price per unit, $x$ be the original number of units sold, and $k$ be the multiplier for the new sales quantity.

Original selling price = $C + 0.25C = 1.25C$, Profit per unit = $0.25C$, total original profit = $0.25C \cdot x$.

New selling price = $90\%$ of original price = $0.9 \times 1.25C = 1.125C$, new profit per unit = $1.125C - C = 0.125C$, total new profit = $0.125C \cdot (kx)$.

New total profit = Original total profit $\times 1.25$.

Equation: $0.125C \cdot (kx) = 1.25 \times (0.25C \cdot x)$

$0.125k = 1.25 \times 0.25$
$0.125k = 0.3125$
$k = \frac{0.3125}{0.125} = 2.5$

Final Result: $\boxed{2.5}$.

5 （1分）Tom took a rope, folded it once, then cut it at the midpoint, resulting in $3$ pieces. When he folded the rope twice and cut once at the midpoint, it became $5$ pieces. If Tom folds the rope $4$ times and cuts $3$ times, how many pieces will the rope be cut into?

A. $46$ B. $47$ C. $48$ D. $49$ E. $50$

Answer D

Solution To determine the number of pieces, focus on the number of cut points:

Folding once and cutting once: The rope has $2^1 = 2$ layers. Cutting once creates $2 \times 1 = 2$ cut points, resulting in $2 + 1 = 3$ pieces.

Folding twice and cutting once: The rope has $2^2 = 4$ layers. Cutting once creates $4 \times 1 = 4$ cut points, yielding $4 + 1 = 5$ pieces.

General formula: After folding $n$ times, the rope has $2^n$ layers. Cutting $k$ times at the midpoint produces $2^n \times k$ cut points.The total number of pieces is: $\text{Pieces} = (2^n \times k) + 1$

For $4$ folds and $3$ cuts: Layers$= 2^4 = 16$, Cut points$= 16 \times 3 = 48$, Total pieces$= 48 + 1 = 49$

.

Final result: $\boxed{49}$.