MAA AMC AMERICAN MATHEMATICS COMPETITIONS

MAA American Mathematics Competitions
40th Annual

AMC 8

Wednesday, January 22, 2025 through Tuesday, January 28, 2025

INSTRUCTIONS

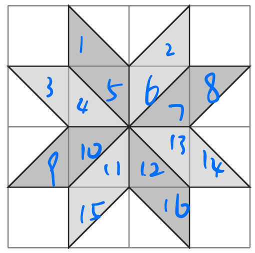
1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 1 point for each correct answer, 0 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, and erasers are allowed as aids. Prohibited materials include calculators, smartwatches, phones, computing devices, compasses, protractors, and graph paper. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. You will have 40 minutes to complete the competition once your competition manager tells you to begin.

The problems and solutions for this AMC 8 were prepared
by the MAA AMC 8 Editorial Board under the direction of:
Silva Chang and Steven Klee

B

1. The eight-pointed star, shown in the figure below, is a popular quilting pattern. What percent of the entire 4-by-4 grid is covered by the star?

$16 \div 2 = 8$ units

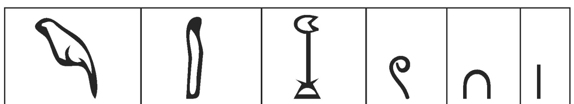


Percent $= \frac{8}{4\times4} = \frac{1}{2} = 50\%$

(A) 40 **(B)** 50 **(C)** 60 **(D)** 75 **(E)** 80

B

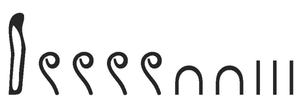
2. The table below shows the ancient Egyptian hieroglyphs that were used to represent different numbers.



100,000	10,000	1,000	100	10	1

For example, the number 32 was represented by ∩∩∩||. What number was represented by the following combination of hieroglyphs?

$10{,}000 + 4\times100 + 2\times10 + 3$



$= 10423$

(A) 1,423 **(B)** 10,423 **(C)** 14,023 **(D)** 14,203 **(E)** 14,230

C

3. *Buffalo Shuffle-o* is a card game in which all the cards are distributed evenly among all players at the start of the game. When Annika and 3 of her friends play *Buffalo Shuffle-o*, each player is dealt 15 cards. Suppose 2 more friends join the next game. How many cards will be dealt to each player?

Total: $(3+1)\times15 = 60$ cards

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

Each player now: $60 \div (3+1+2) = 10$

B

4. Lucius is counting backward by 7s. His first three numbers are 100, 93, and 86. What is his 10th number?

-7 -7

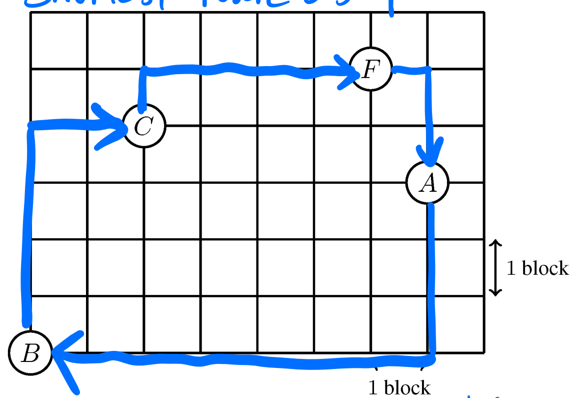
(A) 30 **(B)** 37 **(C)** 42 **(D)** 44 **(E)** 47

$100 - 7\times(10-1) = 37$

C

5. Betty drives a truck to deliver packages in a neighborhood whose street map is shown below. Betty starts at the factory (labeled F) and drives to location A, then B, then C, before returning to F. What is the shortest distance, in blocks, she can drive to complete the route?

Shortest route as follow:



Distance: Regard the whole route as a 7x5 rectangle.
Then $(7\times5)\times2=24$

(A) 20 **(B)** 22 **(C)** 24 **(D)** 26 **(E)** 28

C

6. Sekou writes the numbers 15, 16, 17, 18, 19. After he erases one of the numbers, the sum of the remaining four numbers is a multiple of 4. Which number did he erase?

(A) 15 **(B)** 16 **(C)** 17 **(D)** 18 **(E)** 19

The Sum must be an even number.
Then, the four numbers must be:
2 even ; 2 odd.
One odd number must be erased!
Try 15, 17, 19.
Only by erasing 17, the Sum is $34\times2=68$.

D

7. On the most recent exam in Prof. Xochi's class,

13 students $\geq 90\%$

- 5 students earned a score of at least 95%,
- 13 students earned a score of at least 90%,
- 27 students earned a score of at least 85%, and
- 50 students earned a score of at least 80%.

50 students $\geq 80\%$

How many students earned a score of at least 80% and less than 90%?

(A) 8 **(B)** 14 **(C)** 22 **(D)** 37 **(E)** 45

$50-13=37$ between 80% and 90%.

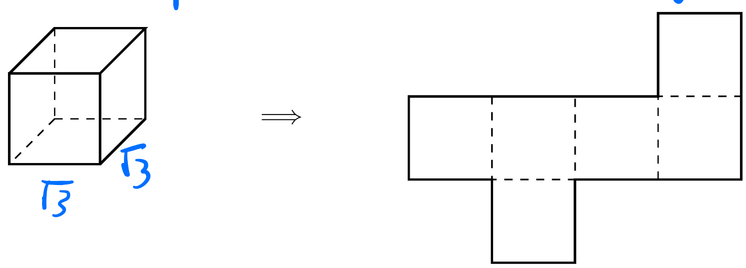
A

8. Isaiah cuts open a cardboard cube along some of its edges to form the flat shape shown on the right, which has an area of 18 square centimeters. What was the volume of the cube in cubic centimeters?

$A_{\text{each face}} = 18 \div 6 = 3$. Edge length $= \sqrt{3}$. $V = (\text{edge length})^3$

$= (\sqrt{3})^3$

$= 3\sqrt{3}$



(A) $3\sqrt{3}$ **(B)** 6 **(C)** 9 **(D)** $6\sqrt{3}$ **(E)** $9\sqrt{3}$

B

9. Ningli looks at the 6 pairs of numbers directly across from each other on a clock. She takes the average of each pair of numbers. What is the average of the resulting 6 numbers?

Step 1:

Step 2:

Step 1: $(1,7) \xrightarrow{\text{Average}} 4$

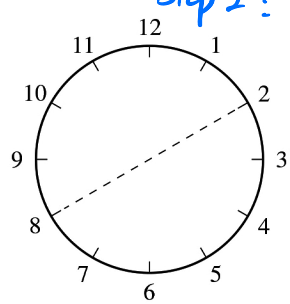
$(2,8) \rightarrow 5$

$(3,9) \rightarrow 6$

$(4,10) \rightarrow 7$

$(5,11) \rightarrow 8$

$(6,12) \rightarrow 9$



Average of 4, 5, 6, 7, 8, 9 is: $(4+5+6+\cdots 9) \div 6 = 6.5$

(A) 5 **(B)** 6.5 **(C)** 8 **(D)** 9.5 **(E)** 12

10. In the figure below, $ABCD$ is a rectangle with sides of length $AB = 5$ inches and $AD = 3$ inches. Rectangle $ABCD$ is rotated $90°$ clockwise around the midpoint of side $\overline{DC}$ to give a second rectangle. What is the total area, in square inches, covered by the two overlapping rectangles?

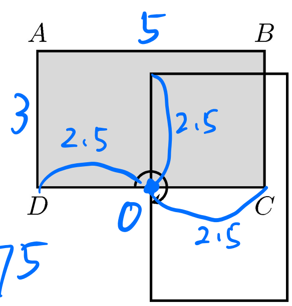
$A_{\text{two rectangles}} - A_{\text{overlapping}}$

$A_{\text{one rectangle}} = 3 \times 5 = 15$

$DO = OC = 2.5$

$A_{\text{overlapping}} = 2.5 \times 2.5 = 6.25$

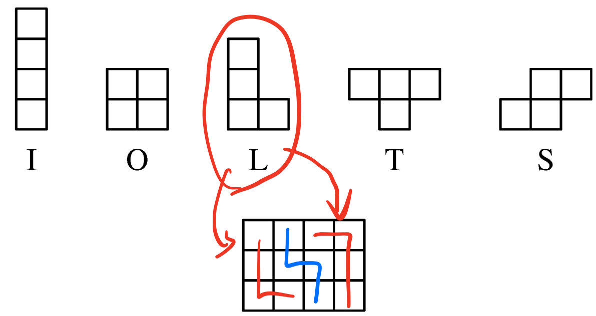
$A_{\text{total area}} = 2 \times 15 - 6.25 = 23.75$



(A) 21 **(B)** 22.25 **(C)** 23 **(D)** 23.75 **(E)** 25

C

11. A *tetromino* consists of four squares connected along their edges. There are five possible tetromino shapes, I, O, L, T, and S, shown below, which can be rotated or flipped over. Three tetrominoes are used to completely cover a 3×4 rectangle. At least one of the tiles is an S tile. What are the other two tiles?

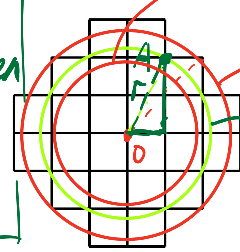


(A) I and L **(B)** I and T **(C)** L and L **(D)** L and S **(E)** O and T

C

12. The region shown below consists of 24 squares, each with side length 1 centimeter. What is the area, in square centimeters, of the largest circle that can fit inside the region, possibly touching the boundaries?

Strategy: Try to find the shortest possible distance between the center to the boundary of the figure.



Classic mistakes ✗

Correct circle ✓

OA is the radius

$r^2 = OA^2 = 1^2 + 2^2 = 5$

$A_{circle} = \pi r^2 = 5\pi$

(A) 3π **(B)** 4π **(C)** 5π **(D)** 6π **(E)** 8π

	2	4	6	8	10	12	14	16	18 ⋯	50
R:	2	4	6	1	3	5	0	2	4 ⋯	5

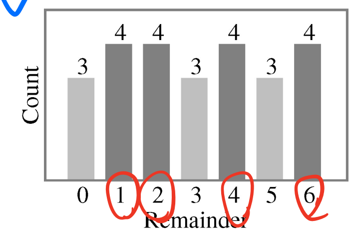
Repeating Cycle !!

A

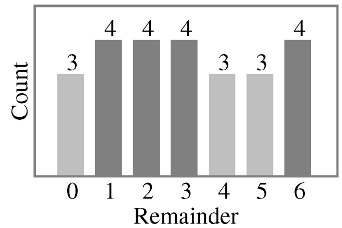
13. Each of the even numbers 2, 4, 6, …, 50 is divided by 7. The remainders are recorded. Which histogram displays the number of times each remainder occurs?

There are 25 numbers in total ∴ $25 \div 7 = 3R4$.

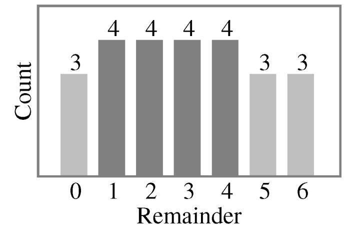
(A)



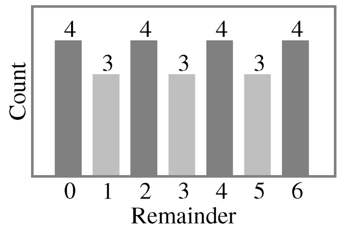
(B)



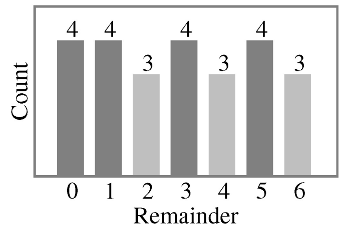
(C)



(D)



(E)



2,4,6,1 each occurs
$3+1=4$ times.
3,5,0 each occurs
3 times.

E

14. A number N is inserted into the list $2, 6, 7, 7, 28$. The mean is now twice as great as the median. What is N?

Median: 7 , Mean: 14

$(14\times5+N)\div6=2\times7 \Rightarrow N=34$

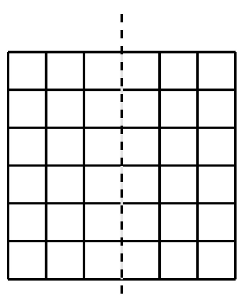
(A) 7 **(B)** 14 **(C)** 20 **(D)** 28 **(E)** 34

From options, we know $N>7$. Then Median must be still 7 after N joins.

C

15. Kei draws a 6-by-6 grid. He colors 13 of the unit squares silver and the remaining squares gold. Kei then folds the grid in half vertically, forming pairs of overlapping unit squares. Let m and M equal the least and greatest possible number of gold-on-gold pairs, respectively. What is the value of $m + M$?

36 units in total: 13 silver, 23 gold



$M: 23\div2=11R1$

$m: (23-13)\div2=5$

(A) 12 **(B)** 14 **(C)** 16 **(D)** 18 **(E)** 20

$m+M=11+5=16$

C

16. Five distinct integers from 1 to 10 are chosen, and five distinct integers from 11 to 20 are chosen. No two numbers differ by exactly 10. What is the sum of the ten chosen numbers?

(A) 95 **(B)** 100 **(C)** 105 **(D)** 110 **(E)** 115

No two numbers have the same $\boxed{\text{ones places}}$.

1~10	11~20
1 2 3 4 5	16 17 18 19 20

Each possible case → has the same sum:
$(1+2+\cdots+10)+5\times10$

6 7 8 9 10 11 12 13 14 15 =105

D

17. In the land of Markovia, there are three cities: A, B, and C. There are 100 people who live in A, 120 who live in B, and 160 who live in C. Everyone works in one of the three cities, and a person may work in the same city where they live. In the figure below, an arrow pointing from one city to another is labeled with the fraction of people living in the first city who work in the second city. (For example, $\frac{1}{4}$ of the people who live in A work in B.) How many people work in A?

For people working in A:

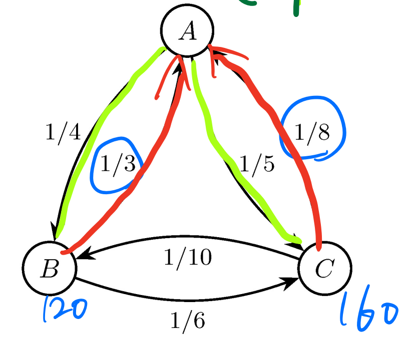
$B \to A: 120 \times \frac{1}{3} = 40$

$C \to A: 160 \times \frac{1}{8} = 20$

$A \to A: 100 \times \frac{11}{20} = 55$

Total: $40 + 20 + 55 = 115$

100 (Left with $1 - \frac{1}{4} - \frac{1}{5} = \frac{11}{20}$ people live & work in A)

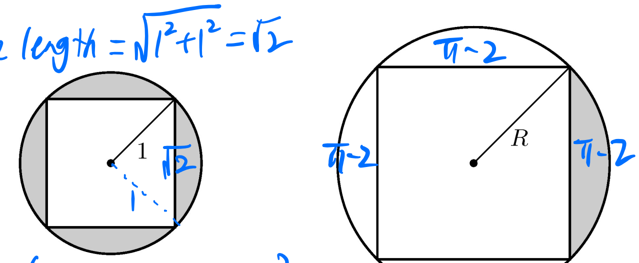


(A) 55 **(B)** 60 **(C)** 85 **(D)** 115 **(E)** 160

B

18. The circle shown below on the left has a radius of 1 unit. The region between the circle and the inscribed square is shaded. In the circle shown on the right, one quarter of the region between the circle and the inscribed square is shaded. The shaded regions in the two circles have the same area. What is the radius R, in units, of the circle on the right?

Side length $= \sqrt{1^2+1^2} = \sqrt{2}$



$A_{shaded} = \pi - \sqrt{2}^2 = \pi - 2 \to A_{shaded} = \pi - 2$

(A) $\sqrt{2}$ **(B)** 2 **(C)** $2\sqrt{2}$ **(D)** 4 **(E)** $4\sqrt{2}$

$A_{circle} : A_{square} = \pi : 2$

Amount Difference: $4 \times (\pi - 2)$

Unit Difference: $\pi - 2$

1 unit $= 4(\pi-2) \div (\pi - 2) = 4$

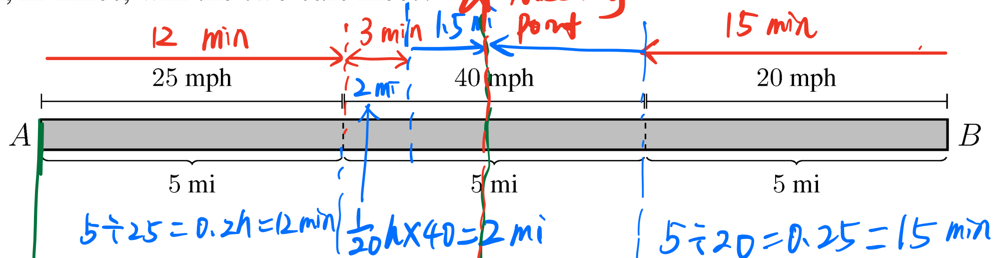
$A_{circle} = 4 \times \pi = 4\pi$

$R = 2$

D

19. Two towns, A and B, are connected by a straight road, 15 miles long. Traveling from town A to town B, the speed limit changes every 5 miles: from 25 to 40 to 20 miles per hour (mph). Two cars, one at town A and one at town B, start moving toward each other at the same time. They drive at exactly the speed limit in each portion of the road. How far from town A, in miles, will the two cars meet?

Meeting point



(A) 7.75 **(B)** 8 **(C)** 8.25 **(D)** 8.5 **(E)** 8.75

$5 + 2 + 1.5 = 8.5$ mi

A

20. Sarika, Dev, and Rajiv are sharing a large block of cheese. They take turns cutting off half of what remains and eating it: first Sarika eats half of the cheese, then Dev eats half of the remaining half, then Rajiv eats half of what remains, then back to Sarika, and so on. They stop when the cheese is too small to see. About what fraction of the original block of cheese does Sarika eat in total?

(A) $\frac{4}{7}$ **(B)** $\frac{3}{5}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** $\frac{7}{8}$

$S = \frac{1}{2} + \frac{1}{16} + \frac{1}{128} \cdots$ 4 units

$D = \frac{1}{4} + \frac{1}{32} + \frac{1}{256} \cdots$ $\times 2$ 2 units $\to \frac{4}{7}$

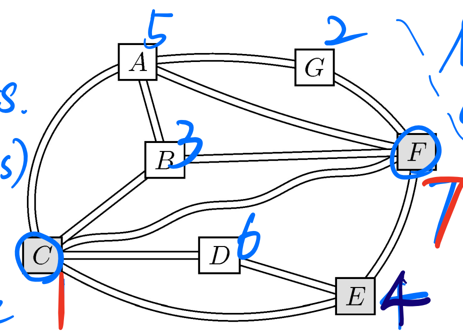
$R = \frac{1}{8} + \frac{1}{64} + \cdots$ $\times 4$ 1 unit

A

21. The Konigsberg School has assigned grades 1 through 7 to pods A through G, one grade per pod. Some of the pods are connected by walkways, as shown in the figure below. The school noticed that each pair of connected pods has been assigned grades differing by 2 or more grade levels. (For example, grades 1 and 2 will not be in pods directly connected by a walkway.) What is the sum of the grade levels assigned to pods C, E, and F?

Numbers which are connected are not consecutive.

Strategy:
C and F are the most connected points. (With more restrictions)
So, try C and F to be 1 and 7 because they are the only two numbers which only have 1 adjacent number.



(A) 12 **(B)** 13 **(C)** 14 **(D)** 15 **(E)** 16

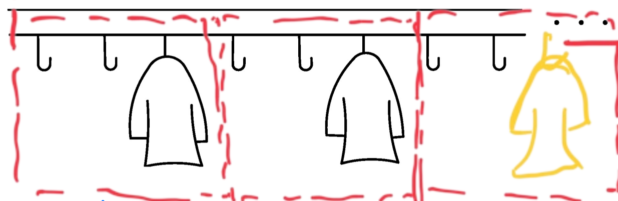
C & F : 1 & 7
D & G : 2 & 6
Left : 3, 4, 5 for A, B, E
But 4 cannot be A or B, otherwise A & B are consecutive numbers.
So, A & B : 3 & 5. → E = 4. C + E + F = 1 + 4 + 7 = 12.

D

22. A classroom has a row of 35 coat hooks. Paulina likes coats to be equally spaced, so that there is the same number of empty hooks before the first coat, after the last coat, and between every coat and the next one. Suppose there is at least 1 coat and at least 1 empty hook. How many different numbers of coats can satisfy Paulina's pattern?

Strategy: We can group the spaces before the coat and the coat together. Then, the goal is getting the number of possible groups:

$\boxed{\text{number of factors}}$



For the last group, we can pretend there is one more coat at last. Then, we have $35+1=36$ hooks totally.

(A) 2 **(B)** 4 **(C)** 5 **(D)** 7 **(E)** 9

$36=2^2\times3^2 \rightarrow (2+1)\cdot(2+1)=9$ factors.

The smallest factor is 1. ✗
largest factor is 36. ✗
Only $9-2=7$ factors. ✓

B

23. How many four-digit numbers have all three of the following properties?

(I) The tens digit and ones digit are both 9.

(II) The number is 1 less than a perfect square.

(III) The number is the product of exactly two prime numbers.

Difference of squares $m^2-1=(m+1)(m-1)$

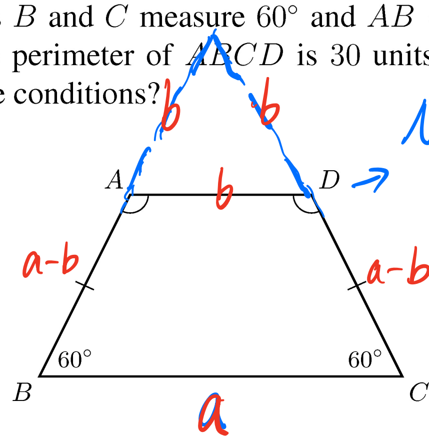
$\square\square99=m^2-1$
$=(m+1)(m-1)$
prime prime

Try: $29\times31=899$ ✗
$59\times61=3599$ ✓

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

E

24. In trapezoid $ABCD$, angles B and C measure 60° and $AB = DC$. The side lengths are all positive integers and the perimeter of $ABCD$ is 30 units. How many non-congruent trapezoids satisfy all of these conditions?



Now it is an equilateral triangle.

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

$a>b$

$3b+a+2(a-b)=3a-b=30$

$3a$ divisible by 3, b divisible by 3

possible value of $3a$
33 → $b=3$ ✓
36 → $b=6$ ✓
39 → $b=9$ ✓
42 → $b=12$ ✓
45 → $b=15$ $a=15$ ✗

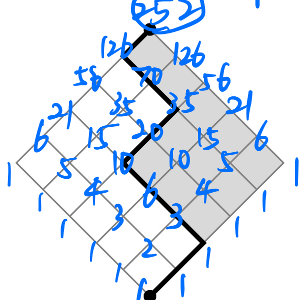
(a must be larger than b)

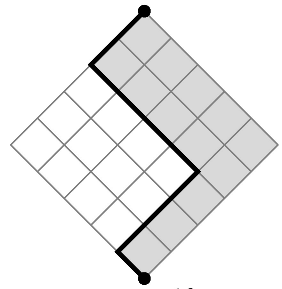
B 25. Makayla finds all the possible ways to draw a path in a 5×5 diamond-shaped grid. Each path starts at the bottom of the grid and ends at the top, always moving one unit northeast or northwest. She computes the area of the region between each path and the right side of the grid. Two examples are shown in the figures below. What is the sum of the areas determined by all possible paths?

Symmetry!

All possible paths can be grouped into pairs in which the sum of the shaded area is $5 \times 5 = 25$

252 # of all possible paths


area = 11


area = 13

So, the sum of all possible area is $(252 \div 2) \times 25 = 3150$

(A) 2520 **(B)** 3150 **(C)** 3840 **(D)** 4730 **(E)** 5050

Problems and Solutions contributed by Hannah Alpert, Helen Beylkin, Owen Byer, Silva Chang, Steven Dunbar, Marta Eso, Thomas Hagedorn, Susan Holtzapple, Steven Klee, Rich Morrow, Amro Mossad, Bryan Nevarez, Jeganathan Sriskandarajah, Patrick Vennebush, and David Wells.