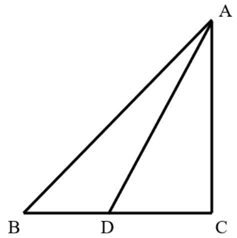
2025 July AMC 10 Week 1&2

2025 July AMC 10 Week 1 Day 1 - Pythagorean Theorem

1 (1分) In $Rt\triangle ABC$, $AB = 4\sqrt{2}$, $AD = 2\sqrt{5}$, and $BD = 2$. What is the length of AC?



A. $\sqrt{6}$ B. 3 C. $2\sqrt{3}$ D. 4 E. $2\sqrt{6}$

Answer D

Solution Suppose $CD = x$, then $BC = BD + CD = 2 + x$.

Using the relationship $AB^2 - BC^2 = AD^2 - CD^2$ (derived from the Pythagorean theorem in $\triangle ABC$ and $\triangle ADC$): $(4\sqrt{2})^2 - (2+x)^2 = (2\sqrt{5})^2 - x^2$

Expand and simplify: $28 - 4x - x^2 = 20 - x^2 \implies 4x = 8 \implies x = 2$

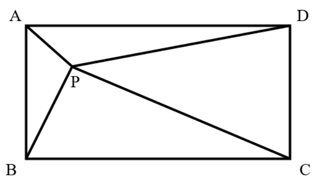
In right triangle $\triangle ADC$, use the Pythagorean theorem:

$AC^2 = AD^2 - CD^2 = (2\sqrt{5})^2 - 2^2 = 20 - 4 = 16$

$AC = \sqrt{16} = 4$

Final Answer: $\boxed{4}$.

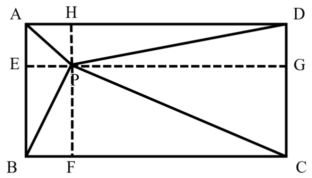
2 (1分) Point P lies inside rectangle $ABCD$. Given $PA = 5$, $PB = 10$, and $PC = 14$, what is the length of PD?



A. 10 B. 11 C. 12 D. 13 E. 14

Answer B

Solution



As shown in the figure, let E、F 、G and$H$ be the projections of P onto AB、BC、CD and DA, respectively.

By the Pythagorean theorem:

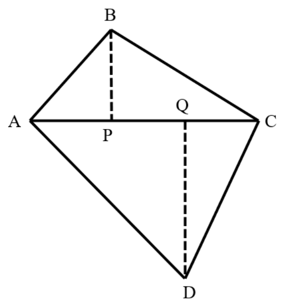
$PA^2 = AE^2 + EP^2, \quad PB^2 = BE^2 + EP^2, \quad PC^2 = CF^2 + FP^2$

Since $ABCD$ is a rectangle, $PD^2 = PA^2 + PC^2 - PB^2 = 25 + 196 - 100 = 121$

Thus, $PD = \sqrt{121} = 11$.

Final Answer: $\boxed{11}$.

3 （1分）In quadrilateral $ABCD$, $AB = 9$, $BC = 12$, $CD = 13$, $DA = 14$, and diagonal $AC = 15$. Perpendiculars are drawn from B and D to AC, intersecting AC at points P and Q respectively. What is the length of PQ?



A. 3 B. 4 C. 5 D. 6 E. 7

Answer A

Solution In $\triangle ACD$, let $AQ = x$. Then $QC = AC - AQ = 15 - x$.

Since $AD^2 - AQ^2 = CD^2 - CQ^2$ (by the Pythagorean theorem in right triangles ADQ and CDQ), we substitute the known values: $14^2 - x^2 = 13^2 - (15 - x)^2$

Expanding and simplifying: $x = \dfrac{42}{5}$

Thus, $AQ = \dfrac{42}{5}$ and $CQ = 15 - \dfrac{42}{5} = \dfrac{33}{5}$.

In $\triangle ABC$, since $AB=9$, $BC=12$, $AC=15$, we check: $9^2+12^2=81+144=225=15^2$.

So $\triangle ABC$ is a right triangle ($\angle ABC=90^\circ$).

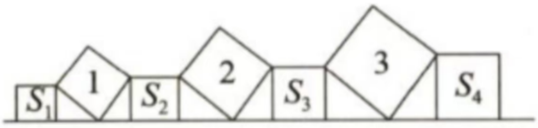
By the geometric mean theorem (or power of a point) in right $\triangle ABC$, $AB^2=AP\cdot AC$.

Substituting values: $9^2=AP\cdot 15 \implies AP=\frac{81}{15}=\frac{27}{5}=5.4$

Thus, $PQ=AC-AP-CQ=15-\frac{27}{5}-\frac{33}{5}=15-\frac{60}{5}=15-12=3$.

Final answer: $\boxed{3}$.

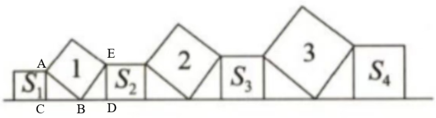
4 (1分) Seven squares are placed in a line in sequence. It is known that the areas of the 3 obliquely placed squares are 1, 2, and 3 respectively, and the areas of the 4 horizontally placed squares are A_1, A_2, A_3, and A_4 respectively. What is the value of $A_1+A_2+A_3+A_4$?



A. 3 B. 4 C. 5 D. 6 E. 7

Answer B

Solution



As shown in the figure, $\angle ACB=\angle BDE=90^\circ$.

Since $\angle ABC+\angle BAC=90^\circ$ and $\angle ABC+\angle EBD=90^\circ$, we can conclude that $\angle BAC=\angle EBD$.

Therefore, $\triangle ABC\cong\triangle BED$ (congruent triangles), so $AC=BD$ and $CB=DE$.

Because $S_1=AC^2$ and $S_2=DE^2$, the area of the first obliquely placed square with side length AB is $AB^2=1$.

By the Pythagorean theorem in $\triangle ABC$, $AB^2=AC^2+CB^2$.

Substituting the equal segments, we get $AB^2=AC^2+DE^2=A_1+A_2$.

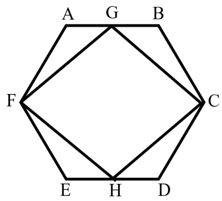
Thus, $A_1+A_2=1$.

Similarly, by observing the horizontally placed squares A_3, A_4 and the middle obliquely placed square, we can obtain $A_3+A_4=3$.

Therefore, $A_1 + A_2 + A_3 + A_4 = 1 + 3 = 4$.

Final answer: $\boxed{4}$.

5 (1分) The side length of regular hexagon $ABCDEF$ is 2, and $AG = BG$, $DH = EH$. What is the perimeter of $GCHG$?



A. 8 B. $4\sqrt{5}$ C. $4\sqrt{6}$ D. $4\sqrt{7}$ E. $8\sqrt{2}$

Answer D

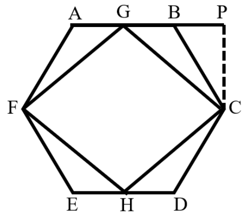
Solution Given that $ABCDEF$ is a regular hexagon, its six sides are all equal, and

$\angle FAG = \angle GBC = \angle CDH = \angle FEH = 120^\circ$.

Since $AG = GB$ and $DH = EH$, triangles $\triangle AFG$, $\triangle BCG$, $\triangle DCH$, and $\triangle EFH$ are congruent.

Thus, quadrilateral $FGCH$ is a rhombus.

Draw CP perpendicular to AB, intersecting the extension of AB at P.



Because $\angle GBC = 120^\circ$, we have $\angle CBP = 60^\circ$, so $BP = 1$ and $CP = \sqrt{3}$.

In $\triangle CGP$, by the Pythagorean theorem: $CG = \sqrt{GP^2 + CP^2} = \sqrt{4+3} = \sqrt{7}$.

Thus, the perimeter of quadrilateral $GFHC$ is $4 \times CG = 4\sqrt{7}$.

2025 July AMC 10 Week 1 Day 2 - "A" and "8" Models

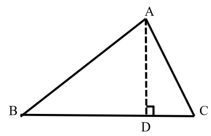
6 (1分)

The altitude of a right-angled triangle divides the hypotenuse into lengths of 4 and 6. What is the area of triangle?

A. 25 B. $10\sqrt{6}$ C. 30 D. $16\sqrt{6}$ E. $20\sqrt{6}$

Answer B

Solution



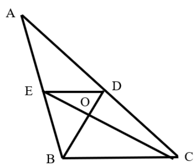
As shown in the figure, since $\angle BAC = \angle ADB = \angle ADC = 90^\circ$, triangles $\triangle ABD$ and $\triangle CAD$ are similar.

Let $AD = h$. By similarity, $\frac{h}{6} = \frac{4}{h}$, thus $h^2 = 24$ and $h = 2\sqrt{6}$.

The area of the triangle is $\frac{1}{2} \times (4+6) \times 2\sqrt{6} = \frac{1}{2} \times 10 \times 2\sqrt{6} = 10\sqrt{6}$.

Final answer: $\boxed{10\sqrt{6}}$.

7 (1分) As shown, in triangle ABC, D and E are the midpoints of AC and AE, respectively. If $BD \perp CE$, $BD = 4$, and $CE = 6$, what is the area of triangle ABC?



A. 12 B. 14 C. 16 D. 18 E. 20

Answer C

Solution Since D and E are midpoints of AC and AE, respectively, $DE \parallel BC$ and $DE = \frac{1}{2}BC$.

Thus, $\triangle ADE \sim \triangle ACB$, so the ratio of their areas is $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $A_{\triangle ADE} : A_{\triangle ACB} = 1 : 4$.

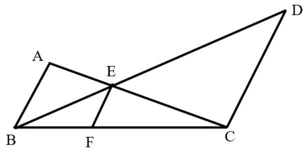
It follows that the area ratio of quadrilateral $BCDE$ to $\triangle ACB$ is $3 : 4$.

$A_{BCDE} = \frac{1}{2} \times BD \times CE = \frac{1}{2} \times 4 \times 6 = 12$.

Then $A_{\triangle ACB} = 16$.

Final answer: $\boxed{16}$.

8 （1分）As shown, $AB \parallel EF \parallel CD$. Given $AC + BD = 240$, $BC = 100$, and $EC + ED = 192$, what is the length of CF?



A. 75 B. 80 C. 82 D. 90 E. 95

Answer B

Solution Since $AB \parallel EF \parallel CD$, we have $\frac{CE}{CF} = \frac{AE}{BF} = \frac{AC}{BC}$ and $\frac{DE}{CF} = \frac{BE}{BF} = \frac{BD}{BC}$.

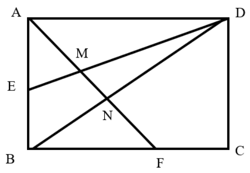
Adding the two equations gives $\frac{CE + DE}{CF} = \frac{AE + BE}{BF} = \frac{AC + BD}{BC}$.

Substituting the known values, we get $\frac{192}{CF} = \frac{240}{100}$.

Solving for CF, we find $CF = 80$.

Final answer: $\boxed{80}$

9 （1分）In rectangle $ABCD$, the side lengths are $AD = 3$ and $AB = 2$. E is the midpoint of AB, and F lies on BC such that $BF = 2FC$. AF intersects DE and DB at points M and N, respectively. What is the length of MN?



A. $\frac{1}{2}$

B. $\frac{3}{10}\sqrt{2}$

C. $\frac{2}{5}\sqrt{2}$

D. $\frac{9}{20}\sqrt{2}$

E. $\frac{1}{2}\sqrt{2}$

Answer D

Solution Since E is the midpoint of AB, we have $AE = EB = 1$.

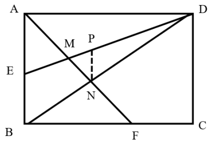
Because $BF = 2FC$, it follows that $BF = 2$ and $FC = 1$.

In $\triangle ABF$, by the Pythagorean theorem, $AF=\sqrt{4+4}=2\sqrt{2}$.

Since $AD \parallel BC$, $\triangle AND \sim \triangle FNB$.

Thus, $NF=\frac{4}{5} sqrt2$ and $AN=\frac{6}{5}\sqrt{2}$.

Draw $NP \parallel AB$ through point N, intersecting DE at point P.

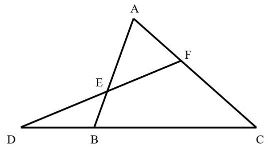


In $\triangle DEB$, $\frac{PN}{EB}=\frac{DN}{DB}=\frac{3}{5}$.

Since $\triangle AME \sim \triangle NMP$, we have $AM:MN=5:3$.

Therefore, $MN=\frac{6}{5}\sqrt{2}\times\frac{3}{8}=\frac{9}{20}\sqrt{2}$.

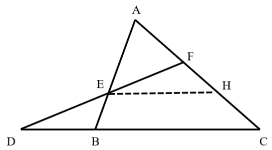
10 (1分) In triangle ABC, point D lies on the extension of BC, and point F lies on AC. Line DF intersects AB at point E. Given $AE:BE=3:2$ and $DE:EF=5:4$, what is the ratio of $AF:FC$?



A. $\frac{1}{4}$
B. $\frac{5}{16}$
C. $\frac{1}{3}$
D. $\frac{7}{18}$
E. $\frac{4}{9}$

Answer D

Solution Draw $EH \parallel BC$ intersecting AC at H.



In $\triangle ABC$, since $AE:BE=3:2$, by the converse of the midpoint theorem:

$\frac{AH}{HC}=\frac{AE}{EB}=\frac{3}{2}$ Let $AH=3x$ and $HC=2x$.

In $\triangle FDC$, since $DE:EF=5:4$, by the intercept theorem:

$$\frac{CH}{FH}=\frac{DE}{EF}=\frac{5}{4} \implies FH=\frac{4}{5}\cdot HC=\frac{4}{5}\cdot 2x=1.6x$$

Calculate AF and FC:

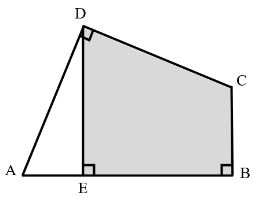
$AF = AH - FH = 3x - 1.6x = 1.4x, \quad FC = FH + HC = 1.6x + 2x = 3.6x$

Therefore, the ratio $AF : FC$ is: $\frac{AF}{FC} = \frac{1.4x}{3.6x} = \frac{14}{36} = \frac{7}{18} \quad \text{(after scaling)}$

Final Answer: $\boxed{\frac{7}{18}}$

2025 July AMC 10 Week 1 Day 3 - "K" Model

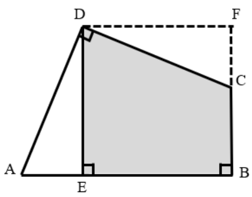
11 (1分) The area of quadrilateral $ABCD$ is 25. It is known that $AD = DC$, $DE = BE$, and $AE = 2$. What is the area of the shaded part?



A. 15 B. 18 C. 19 D. 20 E. 22

Answer D

Solution



Since $AE = CD$, as shown in the figure, draw a line DF through point D perpendicular to the extension of BC at point F.

It can be obtained that $\triangle ADE \cong \triangle CDF$ (congruent triangles).

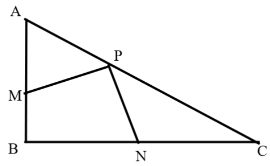
Therefore, quadrilateral $ABCD$ can be transformed into square $DFBE$.

The area of square $DFBE$ is 25, so $DE = EB = 5$.

Then, $BC = BF - CF = BF - AE = 5 - 2 = 3$.

Thus, the area of the shaded part is: $\frac{5 \times (5+3)}{2} = 20$

12 (1分) In right triangle ABC, $AP=2$, $PC=3$, $m\angle C=30^\circ$, $m\angle B=90^\circ$, and $\overline{PM}\perp\overline{PN}$. What is the ratio $\overline{PM}:\overline{PN}$?



A. $\frac{2}{3}$

B. 1

C. $\frac{\sqrt{3}}{3}$

D. $\frac{4}{3}$

E. $\frac{2}{3}\sqrt{3}$

Answer E

Solution Draw $\overline{PE}\perp\overline{AB}$ intersecting $\overline{AB}$ at point H, and draw $\overline{PF}\perp\overline{BC}$ intersecting $\overline{BC}$ at point F

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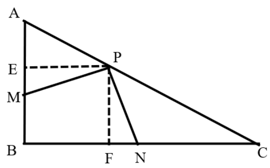


Since $m\angle PEM=m\angle PFN=m\angle ABC=90^\circ$, quadrilateral $PEBF$ is a square.

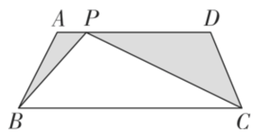
Because $m\angle EPF=m\angle MPN=90^\circ$, we have $m\angle EPM=m\angle FPN$, so right triangles $\triangle EPM$ and $\triangle FPN$ are similar.

Given $m\angle C=30^\circ$, $AP=2$, and $PC=3$, we find $PF=\frac{3}{2}$ and $PE=\sqrt{3}$.

In $\triangle EPM$ and $\triangle FPN$, $\frac{PF}{PE}=\frac{PM}{PN}$, so $\frac{PM}{PN}=\frac{2}{3}\sqrt{3}$.

The final answer is $\frac{2}{3}\sqrt{3}$.

13 (1分) As shown in the figure, in trapezoid $ABCD$, $\overline{AD}\parallel\overline{BC}$, $AD<BC$, $AD=5$, $AB=DC=2$, and P is a point on $\overline{AD}$ such that $m\angle BPC=m\angle A$.What is the length of AP?



A. 1

B. 2

C. 4

D. 1 or 4

E. 2 or 4

Answer D

Solution Since $AB = DC = 2$ in trapezoid $ABCD$, we have $m\angle A = m\angle D$.

Also, because $m\angle BPC = m\angle A$, and $m\angle A + m\angle ABP + m\angle APB = 180^\circ$,

$m\angle APB + m\angle BPC + m\angle CPD = 180^\circ$, it follows that $m\angle ABP = m\angle CPD$.

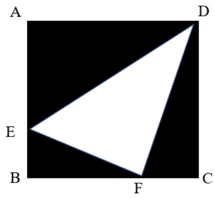
Thus, $\triangle ABP \sim \triangle CPD$, so $\frac{AB}{DP} = \frac{AP}{CD}$.

Let $AP = x$, then $DP = 5 - x$.

Substituting gives $\frac{2}{5-x} = \frac{x}{2}$, solving which yields $x = 1$ or $x = 4$.

Final answer: $\boxed{1 \text{ or } 4}$.

14 (1分) As shown in the figure, $ABCD$ is a square. Given $EF = 3$, $DF = 4$, and $DE = 5$, what is the area of the shaded region?



A. 6

B. $\frac{149}{17}$

C. $\frac{154}{17}$

D. 14

E. $\frac{256}{17}$

Answer C

Solution Since $EF = 3$, $DF = 4$, and $DE = 5$, by the Pythagorean theorem, $\triangle DEF$ is a right triangle with $m\angle DFE = 90^\circ$.

Triangles $\triangle EBF$ and $\triangle FCD$ are similar by AA similarity.

Thus, the ratio of corresponding sides is: $\frac{BF}{CD} = \frac{EF}{FD} = \frac{EB}{FC} = \frac{3}{4}$.

Let $BF = 3x$ and $CD = 4x$. Then $CF = BC - BF = CD - BF = 4x - 3x = x$.

Applying the Pythagorean theorem in $\triangle FCD$:

$$FC^2 + CD^2 = FD^2 \implies x^2 + (4x)^2 = 4^2 \implies 17x^2 = 16 \implies x^2 = \frac{16}{17}$$

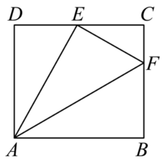
The area of the square $ABCD$ is $(4x)^2 = 16x^2 = 16 \cdot \frac{16}{17} = \frac{256}{17}$.

The area of $\triangle DEF$ is $\frac{3 \times 4}{2} = 6$.

The area of the shaded region is: $\frac{256}{17}-6=\frac{256}{17}-\frac{102}{17}=\frac{154}{17}$

Final Answer: $\boxed{\frac{154}{17}}$

15 (1分) As shown in the figure, in rectangle $ABCD$, $AB=6$. Point B is folded to point E on side CD along crease AF, and AE and EF are connected. If E is the midpoint of CD, what is the length of CF?



A. 1 B. $\sqrt{3}$ C. 2 D. $\sqrt{5}$ E. 3

Answer B

Solution In rectangle $ABCD$, since $AB=6$, we have $CD=6$.

As E is the midpoint of CD, it follows that $DE=CE=3$.

In right triangle ADE, $AD=\sqrt{6^2-3^2}=3\sqrt{3}$.

From the problem, $m\angle D=m\angle C=m\angle AEF=90^\circ$.

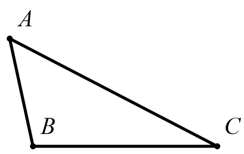
Since $m\angle AED+m\angle CEF=m\angle EFC+m\angle CEF=90^\circ$, we get $m\angle AED=m\angle EFC$, so $\triangle ADE\sim\triangle EFC$.

Thus, $\frac{CF}{DE}=\frac{CE}{DA}$, i.e., $\frac{CF}{3}=\frac{3}{3\sqrt{3}}$, solving which gives $CF=\sqrt{3}$.

Final answer: $\boxed{\sqrt{3}}$.

2025 July AMC 10 Week 2 Day 1 - Properties Triangles

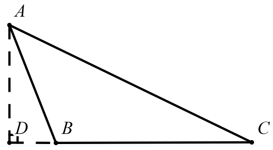
16 (1分) As shown, $AB=3$, $BC=5$, $AC=7$. What is the measure of $\angle ABC$?



A. $96°$ B. $100°$ C. $110°$ D. $120°$ E. $130°$

Answer D

Solution Draw $AD \perp BC$ extending BC to D.



Let $BD = x$, so $CD = x + 5$.

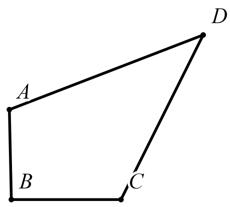
By the Pythagorean theorem: $AB^2 - BD^2 = AC^2 - CD^2 \implies 3^2 - x^2 = 7^2 - (x+5)^2$.

Expanding: $9 - x^2 = 49 - (x^2 + 10x + 25) \implies 10x = 15 \implies x = \frac{3}{2}$.

In $\triangle ABD$, $BD = \frac{3}{2} = \frac{1}{2}AB$, so $\angle BAD = 30°$, hence $\angle ABD = 60°$.

Thus: $\angle ABC = 180° - 60° = 120°$.

17 (1分) In quadrilateral $ABCD$, $m\angle B = 90°$, $AB = 3$, $BC = 4$, $CD = 7$, $AD = 8$. What is the area of $ABCD$?

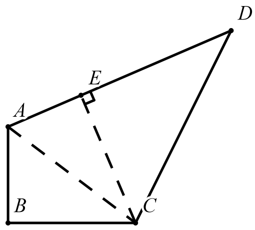


A. 16 B. $6 + 10\sqrt{3}$ C. $12 + 10\sqrt{3}$ D. $6 + 20\sqrt{3}$ E. $20\sqrt{3}$

Answer B

Solution Connect AC. Draw $CE \perp AD$ with E on AD.

In right $\triangle ABC$: $AC = \sqrt{AB^2 + BC^2} = \sqrt{3^2 + 4^2} = 5$.



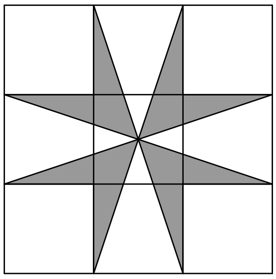
Let $AE = x$, then $DE = 8 - x$.

By Pythagoras: $AC^2 - AE^2 = CD^2 - DE^2 \implies 5^2 - x^2 = 7^2 - (8-x)^2$.

Solving gives $x = 2.5$, so $CE = \sqrt{5^2 - 2.5^2} = \frac{5\sqrt{3}}{2}$.

Total area: $\text{Area of } \triangle ABC + \text{Area of } \triangle ACD = \frac{1}{2} \times 3 \times 4 + \frac{1}{2} \times 8 \times \frac{5\sqrt{3}}{2} = 6 + 10\sqrt{3}$.

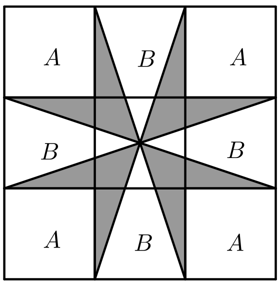
18 (1分) In a 3×3 square grid (small square side $= 1$), what is the area of the shaded region?



A. 1 B. 1.5 C. 2 D. 2.5 E. 3

Answer C

Solution

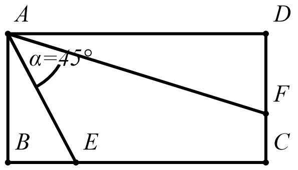


Since the area of the square grid is $3 \times 3 = 9$, and the blank part consists of 4 squares with side length 1 and 4 triangles, the area of the blank part is:

$1 \times 1 \times 4 + 1 \times (3 \div 2) \div 2 \times 4 = 7$.

Thus, the area of the shaded part is: $9 - 7 = 2$.

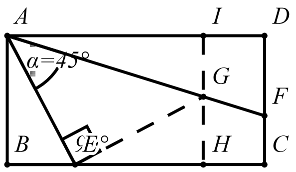
19 (1分) In rectangle $ABCD$, $AB=4$, $AD=8$, $BE=2$, and $\angle EAF=45^\circ$. What is the length of DF?



A. $\frac{7}{3}$ B. $\frac{8}{3}$ C. 3 D. $\frac{7}{2}$ E. $\frac{10}{3}$

Answer B

Solution Draw $EG\perp AE$ intersecting AF at G, then draw $GH\perp BC$ (with H on BC) and $GI\perp AD$ (with I on AD).



Since $\angle EAF=45^\circ$ and $\angle AEG=90^\circ$, $\triangle AEG$ is isosceles right-angled, $AE=EG$.

Calculate AE using the Pythagorean theorem in $\triangle ABE$:

$AE=\sqrt{AB^2+BE^2}=\sqrt{4^2+2^2}=\sqrt{20}$.

By angle relationships: $\angle BAE=\angle GEH$ and $\angle AEB=\angle EGH$ (since their sums with intermediate angles equal 90°).

With $AE=EG$, $\triangle ABE\cong\triangle EHG$ (AAS congruence).

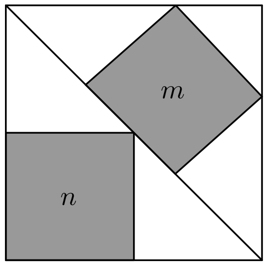
Thus: $EH=AB=4,\quad GH=BE=2$.

This gives $AI=BE+EH=2+4=6$ and $GI=GH=2$.

Since $\triangle AIG\sim\triangle ADF$ (both right-angled and sharing $\angle DAF$):

$$\frac{AI}{AD}=\frac{IG}{DF}\implies\frac{6}{8}=\frac{2}{DF}\implies DF=\frac{8}{3}.$$

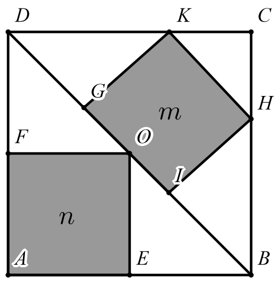
20 (1分) A large square contains two smaller squares with areas m and n. What is the value of $\frac{m}{n}$?



A. $\frac{2}{3}$ B. $\frac{8}{9}$ C. 1 D. $\frac{9}{8}$ E. $\frac{4}{3}$

Answer B

Solution



As shown in the figure, let $AB = 2a$.

Since $ABCD$ is a square, we have: $AD = BC = CD = AB = 2a$, $\angle ABC = \angle ADC = 90^\circ$.

Thus: $\angle ABD = \angle CBD = \frac{1}{2}\angle ABC = 45^\circ$, $\angle BDC = \frac{1}{2}\angle ADC = 45^\circ$.

The length of the diagonal BD of square $ABCD$ is calculated using the Pythagorean theorem: $BD = \sqrt{(2a)^2 + (2a)^2} = \sqrt{8a^2} = 2\sqrt{2}a$.

For the square $AEFO$ with area n:

Due to the 45° angles in the square, $OE = BE = a$.

The side length of this square is a, so its area is: $n = a^2$.

For the square $GKHI$ with area m:

Since $GEHI$ is a square, $HI = GK$, which implies $DG = GI = BI$.

Thus: $GI = \frac{BD}{3} = \frac{2\sqrt{2}a}{3}$.

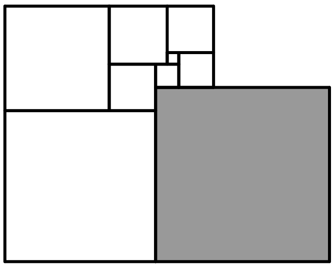
The side length of square $GEHI$ is GI, so its area is: $m = \left(\frac{2\sqrt{2}a}{3}\right)^2 = \frac{8a^2}{9}$.

The ratio of the areas is therefore: $\frac{m}{n} = \frac{\frac{8a^2}{9}}{a^2} = \frac{8}{9}$.

Final answer: $\boxed{\frac{8}{9}}$.

2025 July AMC 10 Week 2 Day 2 - Polygons

21 （1分）The figure below is composed of 9 squares. Given that the area of the smallest square is 1 square centimeter, what is the area of the shaded square?



A. $196\mathrm{cm}^2$

B. $225\mathrm{cm}^2$

C. $256\mathrm{cm}^2$

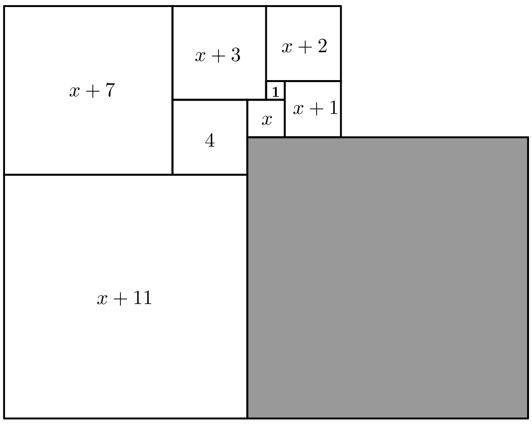
D. $289\mathrm{cm}^2$

E. $324\mathrm{cm}^2$

Answer B

Solution Let the side length of the square below the unit square be x.

The side lengths of the 9 squares can be labeled as shown in the figure.



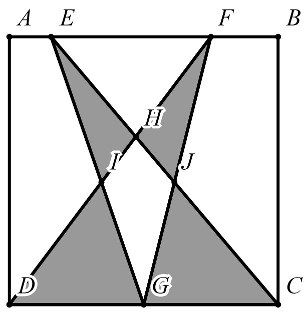
Through analysis, we derive the relationship: $4+(x+11)-x=15$.

This indicates that the side length of the shaded square is 15 centimeters.

Therefore, the area of the shaded square is: $15^2=225\mathrm{cm}^2$.

Answer: The area of the shaded square is 225 square centimeters.

22 （1分）As shown in the figure, quadrilateral $ABCD$ is a square with side length 8, and the area of quadrilateral $GIHJ$ is 5. What is the area of the shaded part in the figure?



A. 22　　B. 24　　C. 26　　D. 28　　E. 30

Answer A

Solution The shaded part in the figure consists of 4 triangles with unknown bases and heights, so their areas cannot be directly calculated.

If the quadrilateral $GIHJ$ is included, then：

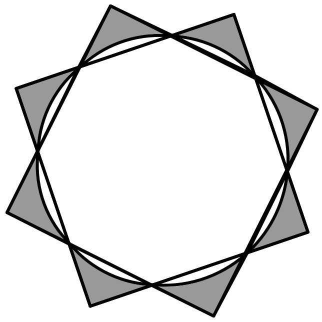
Area of the shaded part = Area of $\triangle DFG +$ Area of $\triangle CEG$ $-2\times$ Area of $GIHJ$.

Therefore: $A_{\triangle DFG}+A_{\triangle CEG}=\frac{1}{2}\times CG\times 8+\frac{1}{2}\times DG\times 8=\frac{1}{2}\times(CG+DG)\times 8$.

Since $CG+DG=CD=8$, we have: $A_{\triangle DFG}+A_{\triangle CEG}=\frac{1}{2}\times 8\times 8=32$.

Thus, the area of the shaded part is: $\text{Area of shaded part}=32-2\times 5=22$.

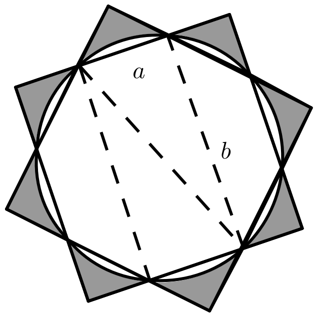
23 （1分）The pattern in the figure below consists of 1 circle and 2 squares of the same size (the common part of the two squares is a regular octagon). If the radius of the circle is $60cm$, then what is the area of the shaded part? (Take $\pi=3.14$)



A. $2096cm^2$ B. $3096cm^2$ C. $4096cm^2$ D. $5096cm^2$ E. $6096cm^2$

Answer B

Solution As shown in the figure below, let the hypotenuse of the small right triangle be a and the side length of the large square be b.



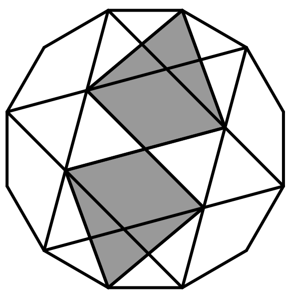
According to the Pythagorean theorem: $a^2+b^2=120^2=14400$.

Here, b^2 represents the area of the large square, and a^2 represents the area of 4 small right triangles, which together form the total area of the figure below.

Thus, the total area is $14400cm^2$.

Therefore, the area of the shaded part is: $14400-3.14\times 60^2=3096cm^2$.

24 （1分）As shown in the figure, the area of a regular dodecagon is $2022cm^2$. What is the area of the shaded part in the figure?



A. $504cm^2$ B. $568cm^2$ C. $612cm^2$ D. $674cm^2$ E. $760cm^2$

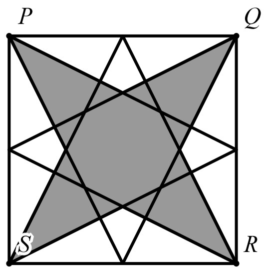
Answer D

Solution By observing the figure, it is found that the regular dodecagon is composed of 12 equilateral triangles and 6 quadrilaterals.

The shaded part is composed of 4 equilateral triangles and 2 quadrilaterals, which is exactly one-third of the regular dodecagon.

Therefore, the area of the shaded part in the figure is: $\frac{1}{3} \times 2022 = 674$.

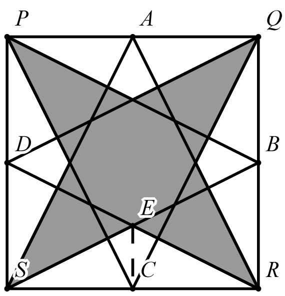
25 （1分）Within the square $PQRS$, lines are drawn from each corner to the middle of the opposite sides as shown. What fraction of $PQRS$ is shaded?



A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{3}{8}$ D. $\frac{1}{2}$ E. $\frac{2}{3}$

Answer D

Solution As shown in the figure, connect CE.



It is easy to see that $\triangle REC$ is similar to $\triangle RDS$, so $CE = \frac{1}{2}DS = \frac{1}{4}PS$.

Let the side length of square $PQRS$ be a.

Then the area of $\triangle ESR$ is $\frac{1}{4}a \times a \div 2 = \frac{1}{8}a^2$.

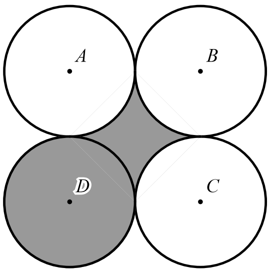
Thus, the area of the blank part is $4 \times \frac{1}{8}a^2 = \frac{1}{2}a^2$.

Therefore, Area of the shaded part = Area of square $PQRS$ − Area of the blank part $= \frac{1}{2}a^2$.

So the fraction of $PQRS$ that is shaded is $\frac{1}{2}$.

2025 July AMC 10 Week 2 Day 3 - Circles

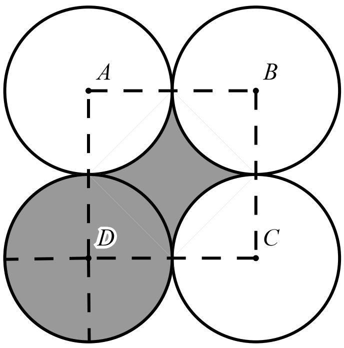
26 (1分) As shown in the figure, it is known that the radius of each of the four circles is 2. What is the area of the shaded part?



A. 4 B. 12 C. 4π D. 16 E. 8π

Answer D

Solution



Connect the centers of the four circles, we find that a square $ABCD$ is formed, with a side length of $2r$.

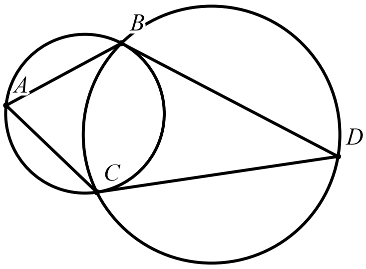
Square $ABCD$ is composed of four sectors and a central figure.

The shaded part is exactly a circle and the central figure, which is also equal to the sum of the four sectors and the central figure.

Therefore, the area of the shaded part is equal to the area of square $ABCD$:

$\text{Area of shaded part} = 2r \times 2r = 2 \times 2 \times 2 \times 2 = 16$.

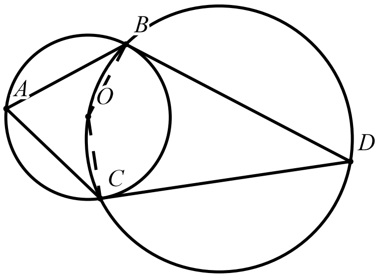
27 (1分) Circle BCD passes through the center of circle ABC. If $\angle BDC = 40^\circ$, and $\angle BAC = a^\circ$, what is the value of a?



A. 40 B. 50 C. 60 D. 70 E. 80

Answer D

Solution Let O be the center of Circle ABC, and connect OB and OC.



In Circle BCD, the two circumferential angles subtended by chord BC are $\angle BDC$ and $\angle BOC$ respectively.

Then: $\angle BDC + \angle BOC = 180^\circ$.

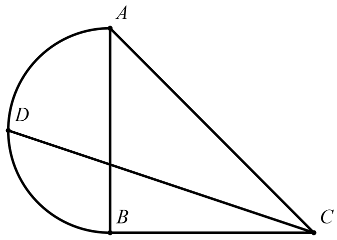
That is: $\angle BOC = 180^\circ - \angle BDC = 180^\circ - 40^\circ = 140^\circ$.

In Circle ABC, the circumferential angle subtended by arc BC is $\angle BAC$, and the central angle subtended by arc BC is $\angle BOC$.

Therefore: $\angle BAC = \frac{1}{2}\angle BOC = \frac{1}{2} \times 140^\circ = 70^\circ$.

Thus, $a = 70$.

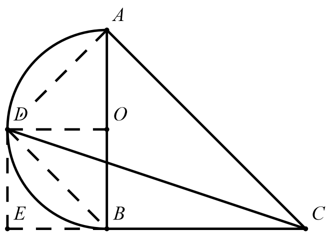
28 (1分) $\triangle ABC$ is an isosceles right-angled triangle where $\angle ABC = 90^\circ$ and $AC = 20$. Construct a semicircle outside $\triangle ABC$ using AB as the diameter. D is a point on the semicircle such that $AD = BD$. What is the length of CD?



A. $12\sqrt{2}$ B. $10\sqrt{5}$ C. 25 D. $15\sqrt{2}$ E. $15\sqrt{5}$

Answer B

Solution As shown in the figure, take the midpoint of AB as point O, connect OD、AD、BD, and draw $DE \perp$ the extension of CB with the foot of the perpendicular being E.



Since point D lies on the semicircle with AB as the diameter, $\angle ADB = 90^\circ$.

Since $AD = BD$, $\triangle ABD$ is an isosceles right-angled triangle.

Since O is the midpoint of AB, $OD \perp AB$ and $OD = OA = OB$.

Because $\angle ABC = 90^\circ$ and $DE \perp BC$, we have $\angle DEB = \angle EBO = \angle DOB = 90^\circ$, so quadrilateral $OBED$ is a square.

Since $\triangle ABC$ is an isosceles right-angled triangle with $AC = 20$, so

$$AB = BC = \frac{AC}{\sqrt{2}} = \frac{20}{\sqrt{2}} = 10\sqrt{2}.$$

Thus, $OA = OB = DE = BE = 5\sqrt{2}$.

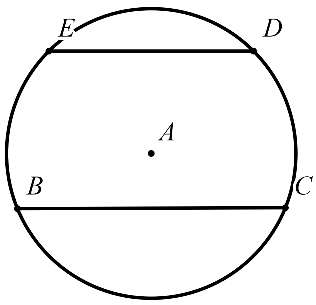
In right triangle CDE, $CE = BE + BC = 5\sqrt{2} + 10\sqrt{2} = 15\sqrt{2}$.

By the Pythagorean theorem:

$$CD = \sqrt{CE^2 + DE^2} = \sqrt{(15\sqrt{2})^2 + (5\sqrt{2})^2} = \sqrt{450 + 50} = \sqrt{500} = 10\sqrt{5}.$$

Thus, the length of CD is $10\sqrt{5}$.

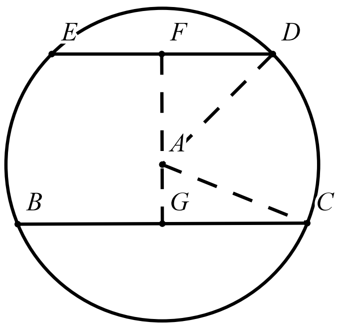
29 （1分）The distance between two parallel chords inside a circle is 8, and the lengths of the chords are 14 and 18 respectively. What is the area of the circle? (Take $\pi = 3.14$)



A. 251.2 B. 254.34 C. 257.48 D. 260.62 E. 266.9

Answer E

Solution Draw $AF \perp ED$ with F on ED and $AG \perp BC$ with G on BC, then connect AD and AC.



According to the perpendicular chord theorem, F and G are the midpoints of ED and BC respectively.

Let the radius of circle A be r, and let $FA = x$, then $AG = 8 - x$.

In right triangle DFA, by the Pythagorean theorem: $DF^2 + FA^2 = AD^2$, that is $7^2 + x^2 = r^2$.

In right triangle CGA, by the Pythagorean theorem: $CG^2 + GA^2 = AC^2$, that is $9^2 + (8 - x)^2 = r^2$.

Set the two equations equal to each other: $7^2 + x^2 = 9^2 + (8 - x)^2$.

Solving for x: $49 + x^2 = 81 + 64 - 16x + x^2$

$16x = 81 + 64 - 49$

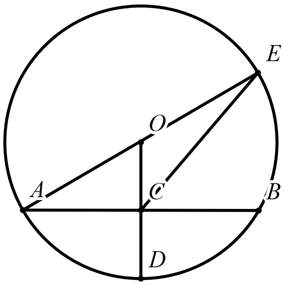
$16x = 96$

$x = 6$

Thus, $r^2 = 7^2 + x^2 = 49 + 36 = 85$.

The area of circle A is: $\pi r^2 = 85 \times 3.14 = 266.9$.

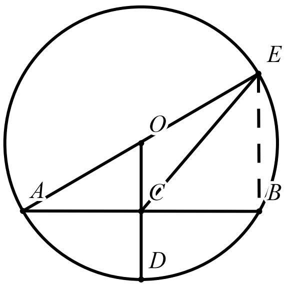
30 （1分）As shown in the figure, the radius OD of circle O is perpendicular to chord AB at point C. Extend AO to intersect circle O at point E, and connect EC. If $AB = 8$ and $CD = 2$, what is the length of EC?



A. $\sqrt{13}$　　B. 6　　C. $4\sqrt{3}$　　D. $5\sqrt{2}$　　E. $2\sqrt{13}$

Answer E

Solution Connect BE.



Let the radius of circle O be R.

Since $OD \perp AB$, by the perpendicular chord theorem: $AC = BC = \frac{1}{2}AB = \frac{1}{2} \times 8 = 4$.

In right triangle AOC, $OA = R$, and $OC = R - CD = R - 2$.

By the Pythagorean theorem in right triangle AOC: $OC^2 + AC^2 = OA^2$.

Substituting the known values: $(R-2)^2 + 4^2 = R^2$.

Solving for R: $R^2 - 4R + 4 + 16 = R^2$, $R = 5$.

Thus, $OC = 5 - 2 = 3$.

Since E is on the circle and AE is a diameter, $\angle ABE = 90^\circ = \angle ACO$, $\angle EAB = \angle OAC$.

Therefore, $\triangle EAB \sim \triangle OAC$.

Thus, $\frac{AC}{AB} = \frac{CO}{BE}$.

$BE = 2OC = 6$.

In right triangle BCE, by the Pythagorean theorem:

$CE = \sqrt{BC^2 + BE^2} = \sqrt{4^2 + 6^2} = \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13}$.