MAA AMC
American Mathematics Competitions

MAA American Mathematics Competitions

25th Annual

AMC 10 A

Wednesday, November 8, 2023

INSTRUCTIONS

1. DO NOT TURN TO THE NEXT PAGE UNTIL YOUR COMPETITION MANAGER TELLS YOU TO BEGIN.
2. This is a 25-question multiple-choice competition. For each question, only one answer choice is correct.
3. Mark your answer to each problem on the answer sheet with a #2 pencil. Check blackened answers for accuracy and erase errors completely. Only answers that are properly marked on the answer sheet will be scored.
4. SCORING: You will receive 6 points for each correct answer, 1.5 points for each problem left unanswered, and 0 points for each incorrect answer.
5. Only blank scratch paper, rulers, compasses, and erasers are allowed as aids. No calculators, smartwatches, phones, or computing devices are allowed. No problems on the competition will require the use of a calculator.
6. Figures are not necessarily drawn to scale.
7. Before beginning the competition, your competition manager will ask you to record your name on the answer sheet.
8. You will have 75 minutes to complete the competition once your competition manager tells you to begin.
9. When you finish the competition, sign your name in the space provided on the answer sheet and complete the demographic information questions on the back of the answer sheet.

The problems and solutions for this AMC 10 A were prepared
by the MAA AMC 10/12 Editorial Board under the direction of
Gary Gordon and Carl Yerger, co-Editors-in-Chief.

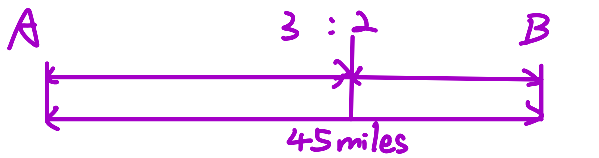
The MAA AMC office reserves the right to disqualify scores from a school if it determines that the rules or the required security procedures were not followed.

The publication, reproduction, or communication of the problems or solutions of this competition during the period when students are eligible to participate seriously jeopardizes the integrity of the results. Dissemination via phone, email, or digital media of any type during this period is a violation of the competition rules.

Students who score well on this AMC 10 will be invited to take the 42nd annual American Invitational Mathematics Examination (AIME) on Thursday, February 1, 2024, or Wednesday, February 7, 2024. More details about the AIME can be found at maa.org/AIME.

1. Cities A and B are 45 miles apart. Alicia lives in A and Beth lives in B. Alicia bikes towards B at 18 miles per hour. Leaving at the same time, Beth bikes toward A at 12 miles per hour. How many miles from City A will they be when they meet?

(A) 20 **(B)** 24 **(C)** 25 **(D)** 26 **(E)** 27



distance = speed × time

$$\frac{\text{distance traveled by A}}{\text{distance traveled by B}} = \frac{\text{speed of A}}{\text{speed of B}} = \frac{18 \text{ mph}}{12 \text{ mph}} = \frac{3}{2}$$

$$\text{distance traveled by A} = 45 \text{ miles} \times \frac{3}{2+3} = 27 \text{ miles}$$

2. The weight of $\frac{1}{3}$ of a large pizza together with $3\frac{1}{2}$ cups of orange slices is the same as the weight of $\frac{3}{4}$ of a large pizza together with $\frac{1}{2}$ cup of orange slices. A cup of orange slices weighs $\frac{1}{4}$ of a pound. What is the weight, in pounds, of a large pizza?

(A) $1\frac{4}{5}$ **(B)** 2 **(C)** $2\frac{2}{5}$ **(D)** 3 **(E)** $3\frac{3}{5}$

Let the weigh of a large pizza be x pounds

$$\frac{x}{3} + 3\frac{1}{2} \times \frac{1}{4} = \frac{3}{4}x + \frac{1}{2} \times \frac{1}{4}$$

$$\frac{7}{2} \times \frac{1}{4} - \frac{1}{2} \times \frac{1}{4} = \left(\frac{3}{4} - \frac{1}{3}\right)x = \left(\frac{3 \times 3}{4 \times 3} - \frac{4}{3 \times 4}\right)x$$

$$\frac{5}{12}x = \frac{3}{4} \Rightarrow x = \frac{9}{5} = 1\frac{4}{5}$$

3. How many positive perfect squares less than 2023 are divisible by 5 ?

(A) 8 **(B)** 9 **(C)** 10 **(D)** 11 **(E)** 12

$45^2 = 2025 > 2023$. so only $5^2, 10^2, \ldots, 40^2$ satisfy.

$$\frac{40-5}{5} + 1 = 8$$

(tips: easy method to calculate the square of number end with 5 $\Rightarrow (10a+5)^2 = 100a(a+1) + 25$

e.g. $45^2 \Rightarrow$ consider $4 \times (4+1) = 20 \Rightarrow$ so $45^2 = 20\underline{25}$

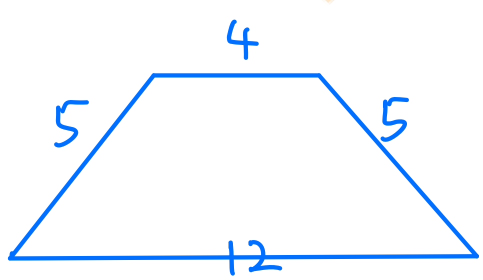
$115^2 \Rightarrow$ consider $11 \times (11+1) = 132 \Rightarrow$ so $115^2 = 13225$

4. A quadrilateral has all integer side lengths, a perimeter of 26, and one side of length 4. What is the greatest possible length of one side of this quadrilateral?

(A) 9 **(B)** 10 **(C)** 11 **(D)** 12 **(E)** 13

for extreme situation, longest side length $< \frac{26}{2} = 13$

so maximum possible value is 12 (because all side lengths are integer)



as figure shown, 12 works.

5. How many digits are in the base-ten representation of $8^5 \cdot 5^{10} \cdot 15^5$?

(A) 14 **(B)** 15 **(C)** 16 **(D)** 17 **(E)** 18

$8^5 \cdot 5^{10} \cdot 15^5 = (2^3)^5 \cdot 5^{10} \cdot (3 \cdot 5)^5$

$= 2^{15} \cdot 5^{10} \cdot 3^5 \cdot 5^5 = 2^{15} \cdot 5^{15} \cdot 3^5$

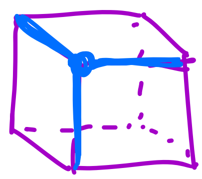
$= 243 \cdot 10^{15}$ $3+15=18$ digits

6. An integer is assigned to each vertex of a cube. The value of an edge is defined to be the sum of the values of the two vertices it touches, and the value of a face is defined to be the sum of the values of the four edges surrounding it. The value of the cube is defined as the sum of the values of its six faces. Suppose the sum of the integers assigned to the vertices is 21. What is the value of the cube?

(A) 42 **(B)** 63 **(C)** 84 **(D)** 126 **(E)** 252

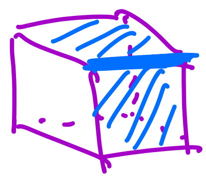
every vertex is calculated three times

(contained in three different edges)



every edge is calculated twice

(contained in two different faces)



so answer is $21 \times 3 \times 2 = 126$

7. Janet rolls a standard 6-sided die 4 times and keeps a running total of the numbers she rolls. What is the probability that at some point, her running total will equal 3?

(A) $\frac{2}{9}$ (B) $\frac{49}{216}$ (C) $\frac{25}{108}$ (D) $\frac{17}{72}$ (E) $\frac{13}{54}$

roll 1 time : $\frac{1}{6}$ (3)

roll 2 times : $\frac{2}{6\times6}$ (1+2, 2+1)

roll 3 times : $\frac{1}{6\times6\times6}$ (1+1+1)

roll 4 times : 0

$\frac{1}{6}+\frac{2}{36}+\frac{1}{216}=\frac{36+12+1}{216}=\frac{49}{216}$

8. Barb the baker has developed a new temperature scale for her bakery called the Breadus scale, which is a linear function of the Fahrenheit scale. Bread rises at 110 degrees Fahrenheit, which is 0 degrees on the Breadus scale. Bread is baked at 350 degrees Fahrenheit, which is 100 degrees on the Breadus scale. Bread is done when its internal temperature is 200 degrees Fahrenheit. What is this in degrees on the Breadus scale?

(A) 33 (B) 34.5 (C) 36 (D) 37.5 (E) 39

Bread	F°
0	110
100	350
x	200

$\frac{350-200}{100-x}=\frac{350-110}{100-0}$

$\frac{150}{100-x}=\frac{240}{100}=\frac{12}{5}$

$100-x=150\times\frac{5}{12}=62.5$

$x=37.5$

(generally, you should assume Bread $= m\cdot F^\circ + b$

after solving $\begin{cases} 0=m\cdot110+b \\ 100=m\cdot350+b \end{cases}$, substitute $F^\circ=200$ to get Bread)

9. A digital display shows the current date as an 8-digit integer consisting of a 4-digit year, followed by a 2-digit month, followed by a 2-digit date within the month. For example, Arbor Day this year is displayed as 20230428. For how many dates in 2023 does each digit appear an even number of times in the 8-digit display for that date?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

for 4 digit month and date, 0 and 3 should appear odd number of times, obviously, they can't appear three times, so both only appear once, that indicate us to enumerate by the position of 3 and 0

~~3 _ _ _~~

0 3 _ _ ⇒ 0311, 0322

0 _ 3 _ ⇒ 0131

_ 0 3 _ ⇒ 1031

_ _ 3 0 ⇒ 1130

0 _ _ 3 ⇒ 0113, 0223

_ 0 _ 3 ⇒ 1013

_ _ 0 3 ⇒ 1103

so there are 9 dates.

10. Maureen is keeping track of the mean of her quiz scores this semester. If Maureen scores an 11 on the next quiz, her mean will increase by 1. If she scores an 11 on each of the next three quizzes, her mean will increase by 2. What is the mean of her quiz scores currently?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

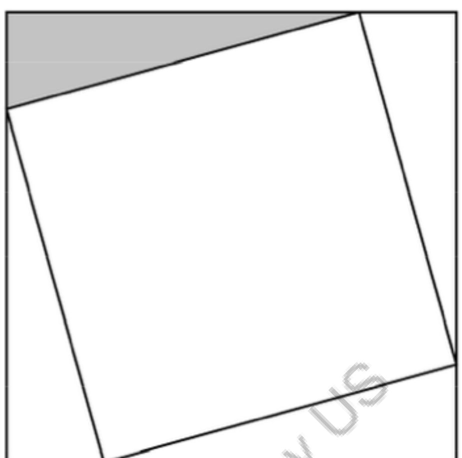
suppose Maureen has taken x quizzes, the total score are y. so we have:

$$\begin{cases} \frac{y+11}{x+1} = \frac{y}{x}+1 \Rightarrow 11 = x+1+\frac{y}{x} & ① \\ \frac{y+11\times 3}{x+3} = \frac{y}{x}+2 \Rightarrow 33 = 2x+6+\frac{3y}{x} & ② \end{cases}$$

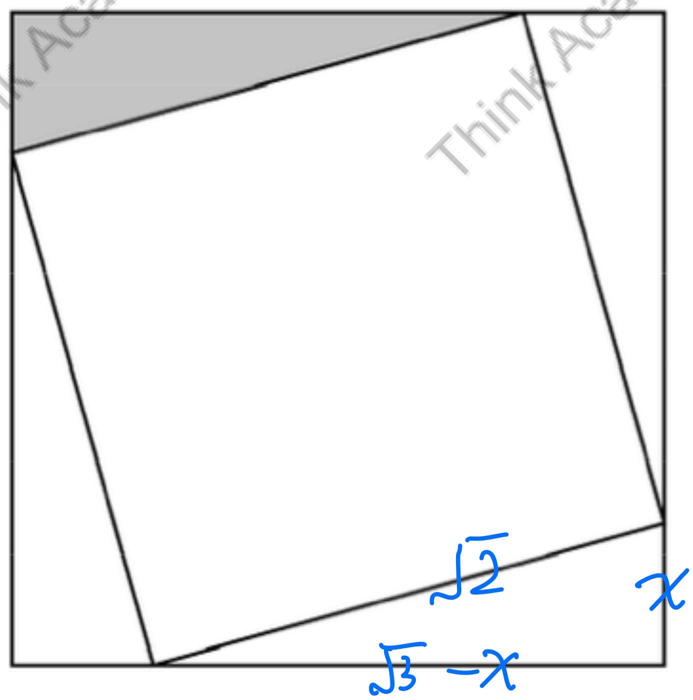
$$② - ①\times 2 \Rightarrow 11 = 4+\frac{y}{x} \Rightarrow \frac{y}{x} = 7.$$

(you can also solve for $\begin{cases} x=3 \\ y=21 \end{cases}$. but you can see that in fact we don't need it)

11. A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle?



(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $2-\sqrt{3}$ (D) $\sqrt{3}-\sqrt{2}$ (E) $\sqrt{2}-1$



this is a simple chord graph, obviously, the four triangles are congruent, assume the shorter leg is x, then the longer leg is $\sqrt{3}-x$, so we have:

$$x^2+(\sqrt{3}-x)^2=(\sqrt{2})^2 \quad \text{(Pythagorean Theorem)}$$

$$\Rightarrow 2x^2-2\sqrt{3}x+1=0$$

by quadratic formula: $x=\dfrac{2\sqrt{3}\pm\sqrt{12-8}}{4}=\dfrac{\sqrt{3}\pm1}{2}$

$\because x<\dfrac{\sqrt{3}}{2}$, $\therefore x=\dfrac{\sqrt{3}-1}{2}$.

$$\therefore \frac{x}{\sqrt{3}-x}=\frac{\frac{\sqrt{3}-1}{2}}{\frac{\sqrt{3}+1}{2}}=\frac{(\sqrt{3}-1)(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}=2-\sqrt{3}$$

12. How many three-digit positive integers N satisfy the following properties?

- The number N is divisible by 7.
- The number formed by reversing the digits of N is divisible by 5.

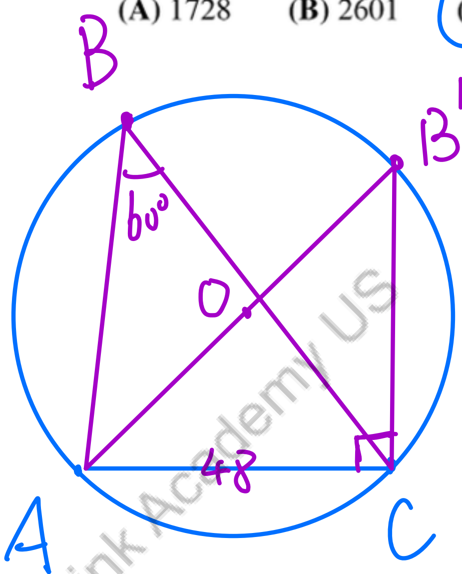
(A) 13 **(B)** 14 **(C)** 15 **(D)** 16 **(E)** 17

by the second property, we know that the leading digit must be 5 (since it can't be 0) so in fact we are looking for the numbers of 7's multiple between 500 – 599

$$\left\lfloor \frac{599}{7} \right\rfloor - \left\lfloor \frac{499}{7} \right\rfloor = 85 - 71 = 14$$

13. Abdul and Chiang are standing 48 feet apart in a field. Bharat is standing in the same field as far from Abdul as possible so that the angle formed by his lines of sight to Abdul and Chiang measures 60°. What is the square of the distance (in feet) between Abdul and Bharat?

(A) 1728 **(B)** 2601 **(C)** 3072 **(D)** 4608 **(E)** 6912



Think about the trajectory of point B (Bharat's possible position)

Because If and only if in the same circle or congruent circles, angles formed by the same arc are equal, so the set of B in fact form an arc as the figure shown.

when AB' max, obviously it will be the diameter. $\triangle AB'C$ is 30-60-90 Rt$\triangle$.

$$\frac{AB'}{AC} = \frac{2}{\sqrt{3}}, \therefore AB' = 32\sqrt{3}, AB'^2 = 3072$$

Solution 2: (Law of Cosine and AM-GM)

$$AB = a,\ BC = b,\ \Rightarrow \frac{a^2+b^2-48^2}{2ab} = \frac{1}{2}$$

$$\Rightarrow a^2 - ab + (b^2 - 48) = 0$$

$$\Rightarrow a = \frac{b \pm \sqrt{b^2 - 4b^2 + 4 \cdot 48}}{2} \leq \frac{b + \sqrt{4 \cdot 48 - 3b^2}}{2}$$

$$= \frac{b + \sqrt{\frac{4 \cdot 48}{9} - \frac{b^2}{3}} + \sqrt{\frac{4 \cdot 48}{9} - \frac{b^2}{3}} + \sqrt{\frac{4 \cdot 48}{9} - \frac{b^2}{3}}}{2}$$

$$\leq 2 \cdot \sqrt{\frac{b^2 + \left(\frac{4 \cdot 48^2}{9} - \frac{b^2}{3}\right) \cdot 3}{4}} = \sqrt{\frac{4 \cdot 48^2}{3}}$$

$$a^2_{max} = \frac{4 \cdot 48^2}{3} = 3072$$

14. A number is chosen at random from among the first 100 positive integers, and a positive integer divisor of that number is then chosen at random. What is the probability that the chosen divisor is divisible by 11 ?

(A) $\frac{4}{100}$ (B) $\frac{9}{200}$ (C) $\frac{1}{20}$ (D) $\frac{11}{200}$ (E) $\frac{3}{50}$

first, the number itself must be a multiple of 11,
it can only be chosen from 11, 22, ..., 99,
the possibility is $\frac{9}{100}$

for each number, since they are all less than 121,
so that means when we split the number $N = ab$,

one and only one among a and b is multiple of 11, so the possibility is $\frac{1}{2}$.

$$\frac{9}{100}\times\frac{1}{2}=\frac{9}{200}$$

15. An even number of circles are nested, starting with a radius of 1 and increasing by 1 each time, all sharing a common point. The region between every other circle is shaded, starting with the region inside the circle of radius 2 but outside the circle of radius 1. An example showing 8 circles is displayed below. What is the least number of circles needed to make the total shaded area at least 2023π ?

(A) 46 **(B)** 48 **(C)** 56 **(D)** 60 **(E)** 64

when we have k circles, the area is:

$$-\pi\cdot1^2+\pi\cdot2^2-\pi\cdot3^2+\cdots+\pi\cdot k^2$$

$$=\pi(-1^2+2^2-3^2+\cdots+k^2)$$

$$=\pi[(2-1)(2+1)+(4-3)(4+3)+\cdots+[k-(k-1)][k+(k-1)]]$$

$$=\pi(1+2+\cdots+k)=\frac{(1+k)k}{2}\pi>2023\pi$$

$$\Rightarrow (1+k)k>4046. \quad \because 64^2=4096>4046$$

$$\therefore k_{min}=64$$

16. In a table tennis tournament every participant played every other participant exactly once. Although there were twice as many right-handed players as left-handed players, the number of games won by left-handed players was 40% more than the number of games won by right-handed players. (There were no ties and no ambidextrous players.) What is the total number of games played?

(A) 15 (B) 36 (C) 45 (D) 48 (E) 66

suppose there are x left-handed players, in total they won y games

so there are $2x$ right-handed players, in total they won $\frac{3x(3x-1)}{2} - y$ games

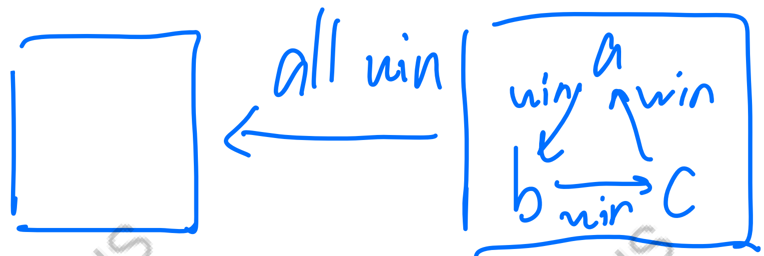
$\therefore y = 1.4\left(\frac{3x(3x-1)}{2} - y\right)$

simplify we get $7x(3x-1) = 8y$

$\therefore 8 \mid x(3x-1)$. when $x=3$, $y=21$

consider

6 Right 3 Left



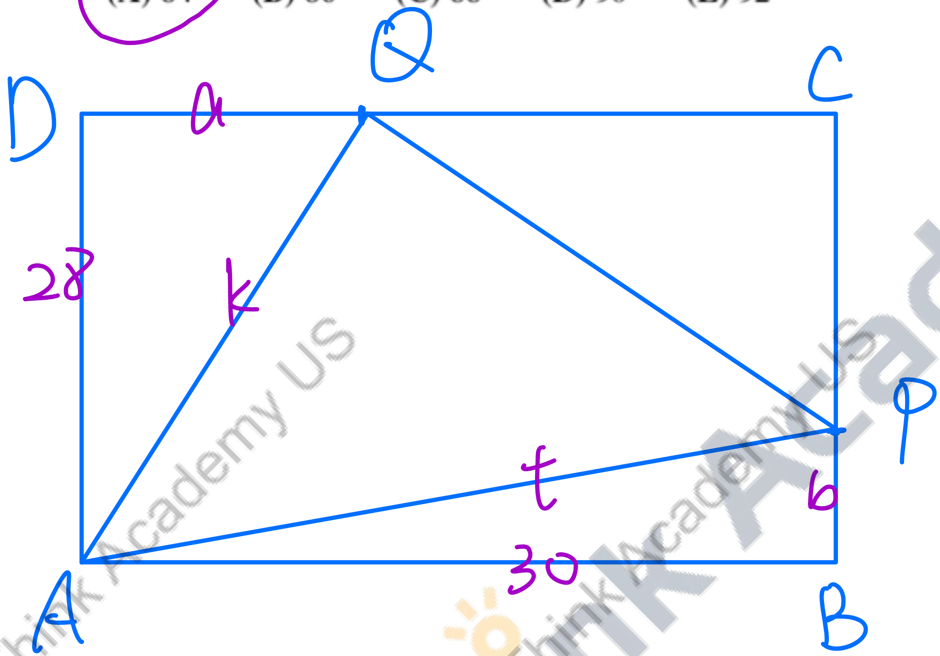
$6 \times 3 + 3 = 21$, works,

$\frac{3x(3x-1)}{2} = \frac{8 \times 9}{2} = 36$.

for other choices, the corresponding x do not satisfy $8 \mid x(3x-1)$

17. Let $ABCD$ be a rectangle with $AB = 30$ and $BC = 28$. Points P and Q lie on $\overline{BC}$ and $\overline{CD}$, respectively, so that all sides of $\triangle ABP$, $\triangle PCQ$, and $\triangle QDA$ have integer lengths. What is the perimeter of $\triangle APQ$?

(A) 84 **(B)** 86 **(C)** 88 **(D)** 90 **(E)** 92



$a < 30$

$b < 28$

$28^2 + a^2 = k^2 \Rightarrow 28^2 = (k+a)(k-a)$.

$\because$ $k+a$, $k-a$ must be even, let $p = \frac{k+a}{2}$, $q = \frac{k-a}{2}$

$\therefore$ we are looking for two integer p, q,

$$\begin{cases} pq = \frac{28^2}{4} = 2^2 \cdot 7^2 \\ 0 < |p-q| = a < 30 \end{cases} \quad \therefore \begin{cases} p = 28 \\ q = 7 \end{cases} \Rightarrow \begin{cases} a = 21 \\ k = 35 \end{cases}$$

similarly, $30^2 + b^2 = t^2 \Rightarrow 30^2 = (t-b)(t+b)$

$\therefore r = \frac{t-b}{2}, \quad s = \frac{t+b}{2}$

$$\begin{cases} rs = \frac{30^2}{2} = 3^2 \cdot 5^2 \\ 0 < |r-s| = b < 30 \end{cases} \quad \therefore \begin{cases} r = 25 \\ s = 9 \end{cases} \Rightarrow \begin{cases} b = 16 \\ t = 34 \end{cases}$$

$$\therefore QP = \sqrt{(30-21)^2 + (28-16)^2} = \sqrt{9^2 + 12^2} = 15$$

$$35 + 34 + 15 = 84$$

Solution 2:

In fact, there are not many Rt$\triangle$ with all three sides as integer. most of them are just a stretch of basic 3-4-5, 5-12-13, Rt$\triangle$s. notice $28 = 4 \times 7$, we guess $\triangle ADQ$ is a 3-4-5 Rt$\triangle$, leads to $QC = 30 - 3 \times 7 = 9 = 3 \times 3$, we also guess

$\triangle QCP$ is a 3-4-5 Rt $\triangle$,

leads to $BP = 28 - 4\times 3 = 16$.

$30^2 + 16^2 = 1156 = 34^2$, it works,

18. A *rhombic dodecahedron* is a convex polyhedron where each of the 12 faces is a rhombus, and all of the faces are congruent to each other. The number of edges that meet at a vertex is either 3 or 4, depending on the vertex. What is the number of vertices at which exactly 3 edges meet?

(A) 5 **(B)** 6 **(C)** 7 **(D)** 8 **(E)** 9

suppose there are x vertices where 3 edges meet. y vertices for 4.

there are $\frac{4\times 12}{2} = 24$ edges

(since rhombus has 4 sides)

$\therefore \frac{3x+4y}{2} = 24$

also, by Euler's formula $(V+F-E=2)$

$x+y = V = 2+24-12$

$\therefore$ solving for $\begin{cases} x=8 \\ y=6 \end{cases}$. $\therefore x=8$.

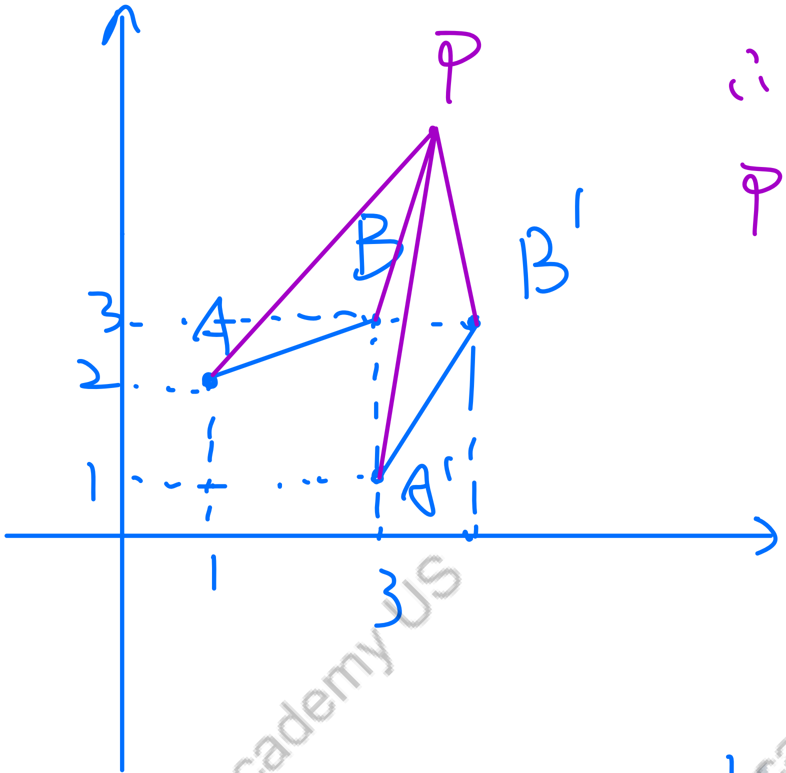
19. The line segment from $A(1,2)$ to $B(3,3)$ can be transformed to the line segment from $A'(3,1)$ to $B'(4,3)$, sending A to A' and B to B', by a rotation centered at the point $P(s,t)$. What is $|s-t|$?

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{2}{3}$ **(D)** $\frac{3}{4}$ **(E)** 1

$\triangle PAB \cong \triangle PA'B'$

$\therefore PA = PA'$, $PB = PB'$

P is on the perpendicular bisector of segment AA' and BB'



the perpendicular bisector of segment BB' is $x = \frac{3+4}{2} = \frac{7}{2}$

line AA': slope $= \frac{1-2}{3-1} = -\frac{1}{2}$.

$\therefore$ slope of perpendicular bisector is $\frac{-1}{-\frac{1}{2}} = 2$

midpoint of AA': $(2, \frac{3}{2})$ $\therefore$ perpendicular bisector of AA': $y = 2x - \frac{5}{2}$ when $s = \frac{7}{2}$, $t = \frac{9}{2}$

$\therefore P(\frac{7}{2}, \frac{9}{2})$. $|s-t| = 1$

20. Each square in a 3 × 3 grid of squares is colored red, white, blue, or green so that every 2 × 2 square contains one square of each color. One such coloring is shown on the right below. How many different colorings are possible?

B	R	B
G		G
R	B	R

(A) 24 (B) 48 (C) 60 (D) 72 (E) 96

B	~~R~~/G	
G/~~R~~	W	

suppose the middle square is white (4 choices)

Top-left square is blue (3 choices, color different from the middle one)

Top-middle and left-middle have 2 different ways to color in total. Now discuss the Top-Right square:

1°

B	R	B
G	W	G
B	R	B
R	B	R

if it is also blue, then Right-middle's color is fixed. bottom row has 2 different ways to color.

2°

B	R	G
G	W	B
B	R	G

if it is not blue, then Right-middle's color is also fixed. bottom row has only 1 ways to color.

in total: $4 \times 3 \times 2 \times (2+1) = 72$

21. Let $P(x)$ be the unique polynomial of minimal degree with the following properties:

- $P(x)$ has leading coefficient 1, $\rightarrow P(1)-1=0$
- 1 is a root of $P(x)-1$, $\rightarrow P(0)=0$
- 2 is a root of $P(x-2)$, $\rightarrow P(9)=0$
- 3 is a root of $P(3x)$, and $\rightarrow P(4)=0$
- 4 is a root of $4P(x)$.

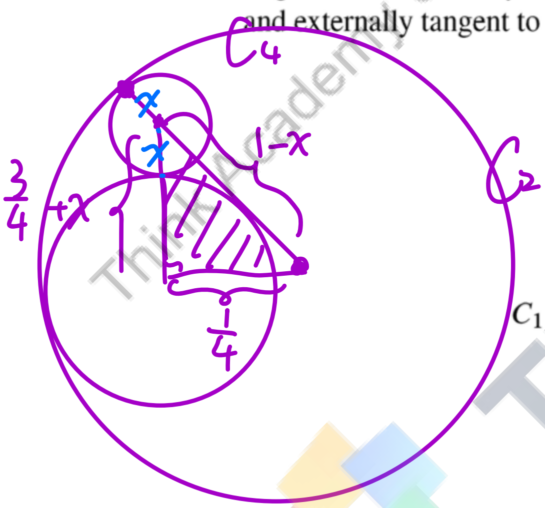
The roots of $P(x)$ are integers, with one exception. The root that is not an integer can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. What is $m+n$?

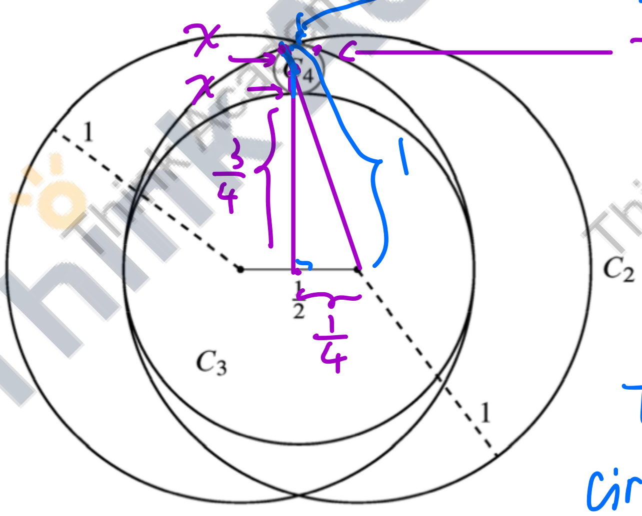
(A) 41 **(B)** 43 **(C)** 45 **(D)** 47 **(E)** 49

by factor theorem, $P(x)=(x-0)(x-9)(x-4)(x-\frac{m}{n})$

$P(1)=24(1-\frac{m}{n})=1 \quad \therefore \frac{m}{n}=\frac{23}{24}, \quad 23+24=47$

22. Circles C_1 and C_2 each have radius 1, and the distance between their centers is $\frac{1}{2}$. Circle C_3 is the largest circle internally tangent to both C_1 and C_2. Circle C_4 is internally tangent to both C_1 and C_2 and externally tangent to C_3. What is the radius of C_4?





not here!

tangent points are here!

easy to get the radius of circle C_3 is $\frac{1+1-\frac{1}{2}}{2}=\frac{3}{4}$

(A) $\frac{1}{14}$ **(B)** $\frac{1}{12}$ **(C)** $\frac{1}{10}$ **(D)** $\frac{3}{28}$ **(E)** $\frac{1}{9}$

Suppose the radius is x,

$(\frac{3}{4}+x)^2+(\frac{1}{4})^2=(1-x)^2 \Rightarrow x=\frac{3}{28}$

tangent point and the two center of circle are ALWAYS collinear!

23. If the positive integer c has positive integer divisors a and b with $c = ab$, then a and b are said to be *complementary* divisors of c. Suppose that N is a positive integer that has one complementary pair of divisors that differ by 20 and another pair of complementary divisors that differ by 23. What is the sum of the digits of N ?

(A) 9 **(B)** 13 **(C)** 15 **(D)** 17 **(E)** 19

suppose $N = a(a+20) = b(b+23)$.

obviously $a > b$, $\because$ a, b are integers,

$\therefore a \geq b+1$ (very important integer handling technique)

$\therefore b(b+23) = a(a+20) \geq (b+1)(b+21)$

$b^2 + 23b \geq b^2 + 22b + 21 \Rightarrow b \geq 21$

when $b = 21$, $21 \times 44 = 22 \times 44 = 924$

$9+2+4 = 15$

Solution 2 (more rigorously)

$a(a+20) = b(b+23) \Rightarrow a^2 - b^2 = 20(b-a) + 3b$

$(a-b)(a+b+20) = 3b$, it obvious that

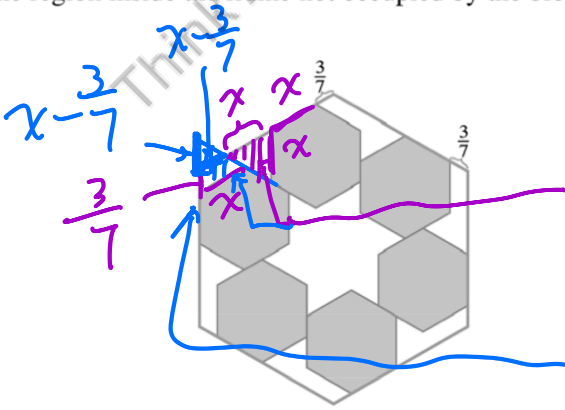
$a-b = 1$ or 2 (when $a-b \geq 3$, easy to find $3(a+b+20) > 3b$).

when $a-b=1$, $a+b+20 = 3b \Rightarrow \begin{cases} a=22 \\ b=21 \end{cases}$

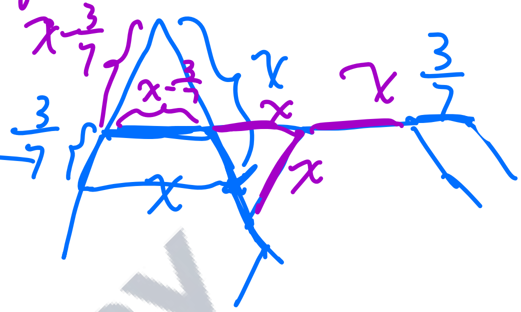
when $a-b=2$, $2(a+b+20) = 3b \Rightarrow \begin{cases} \cancel{a=-42} \\ \cancel{b=-44} \end{cases}$

24. Six regular hexagonal blocks of side length 1 unit are arranged inside a regular hexagonal frame. Each block lies along an inside edge of the frame and is aligned with two other blocks, as shown in the figure below. The distance from any corner of the frame to the nearest vertex of a block is $\frac{3}{7}$ unit. What is the area of the region inside the frame not occupied by the blocks?

area of regular hexagon with side length a is $\frac{3\sqrt{3}}{2}a^2$



equilateral $\triangle$



(A) $\frac{13\sqrt{3}}{3}$ (B) $\frac{216\sqrt{3}}{49}$ (C) $\frac{9\sqrt{3}}{2}$ (D) $\frac{14\sqrt{3}}{3}$ (E) $\frac{243\sqrt{3}}{49}$

Assume the side length of small hexagon is x

$\therefore x-\frac{3}{7}+x+x+\frac{3}{7}=1 \Rightarrow x=\frac{1}{3}$

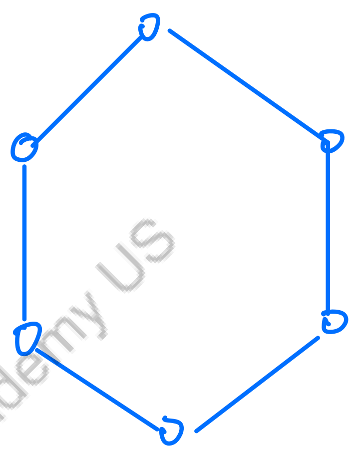
area: $\frac{3\sqrt{3}}{2}\cdot 1^2 - 6\times\frac{3\sqrt{3}}{2}\times\left(\frac{1}{3}\right)^2 = \frac{9\sqrt{3}}{2}$

25. If A and B are vertices of a polyhedron, define the *distance* $d(A, B)$ to be the minimum number of edges of the polyhedron one must traverse in order to connect A and B. For example, if $\overline{AB}$ is an edge of the polyhedron, then $d(A, B) = 1$, but if $\overline{AC}$ and $\overline{CB}$ are edges and $\overline{AB}$ is not an edge, then $d(A, B) = 2$. Let Q, R, and S be randomly chosen distinct vertices of a regular icosahedron (regular polyhedron made up of 20 equilateral triangles). What is the probability that $d(Q, R) > d(R, S)$?

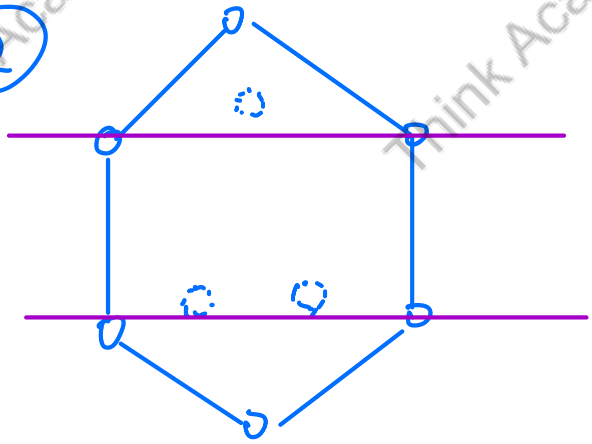
(A) $\frac{7}{22}$ (B) $\frac{1}{3}$ (C) $\frac{3}{8}$ (D) $\frac{5}{12}$ (E) $\frac{1}{2}$

Tips: How to draw a regular icosahedron

① draw a regular hexagon

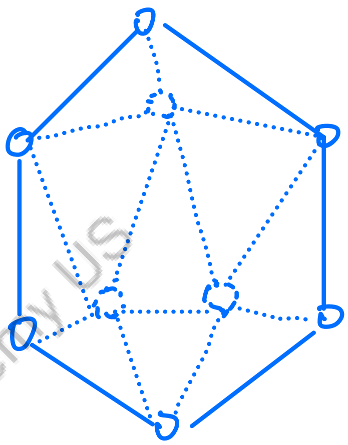


2



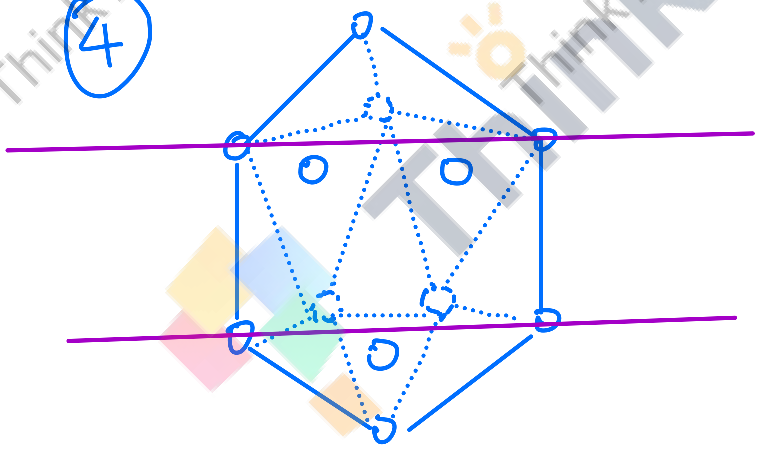
draw three points slightly above the corresponding line

3



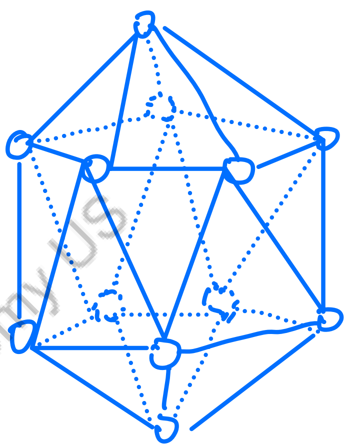
connect to get △

4



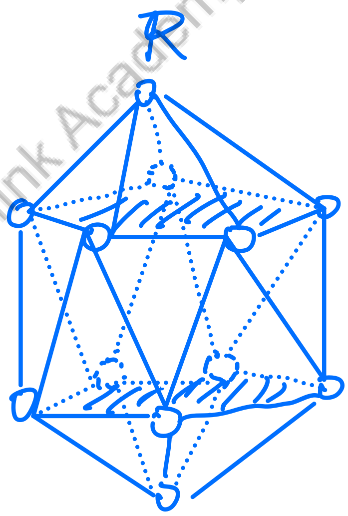
draw three points slightly below the corresponding line (inverted △)

5



connect to get △

done !



Consider R at the Top
(you can always rotate it to get)
there are 5 points make $d(P,R)=1$
5 points make $d(P,R)=2$
1 point make $d(P,R)=3$

when $d(R,S)=1 \Rightarrow \frac{5}{11} \times \frac{5+1}{10}$

$d(R,S)=2 \Rightarrow \frac{5}{11} \times \frac{1}{10}$

$\frac{30}{110} + \frac{5}{110} = \frac{35}{110} = \frac{7}{22}$