

2025 Sept AMC 10 Week 1&2

2025 Sept AMC 10 Week 1 Day 1 - Counting Principles

1 From 4 boys and 2 girls, select 3 people to form a team for the competition. If at least 1 girl must be included, then the total number of different ways to form the team is () .

A. 8

B. 12

C. 16

D. 20

E. 24

Answer C

Solution From the problem, the different selections can be divided into two cases:

Case 1: Exactly 1 girl is selected. The number of different ways is

$${}_2C_1 \times {}_4C_2 = 12.$$

Case 2: Exactly 2 girls are selected. The number of different ways is

$${}_2C_2 \times {}_4C_1 = 4.$$

According to the Addition Principle of Counting, the total number of different selections with at least 1 girl is 16.

Therefore, the answer is C.

2 Given 6 different colors of paint to color the six regions A, B, C, D, E , and F in the figure, with the condition that adjacent regions cannot be painted the same color, how many different colorings are there?

A	C	F
	D	
B	E	

- A. 3880 B. 3180 C. 6120 D. 3240 E. 5180

Answer C

Solution Assuming the coloring order is $D - C - E - F - A - B$,

If C and E are colored the same, then there are

$$6 \times 5 \times 1 \times 4 \times 4 \times 3 = 1440 \text{ ways.}$$

If C and E are different, and A and E are the same, then there are

$$6 \times 5 \times 4 \times 3 \times 1 \times 4 = 1440 \text{ ways.}$$

If C and E are different, and A and E are also different, then there are

$$6 \times 5 \times 4 \times 3 \times 3 \times 3 = 3240 \text{ ways.}$$

Therefore, by the Addition Principle of Counting, the total number of colorings is

$$1440 + 1440 + 3240 = 6120.$$

Hence the answer is **6120**.

- 3** Five students A, B, C, D , and E participate in a certain technical competition and receive the first through fifth places. A and B ask about the results. The organizer tells A : "Unfortunately, neither you nor B won the championship." To B , the organizer says: "Of course, you are not in the last place." Based on the organizer's responses, how many different possible rankings of the five students are there?

- A. 32 B. 38 C. 48 D. 54 E. 64

Answer D

Solution According to the problem, A and B did not win the championship, and B is not in the last place. We discuss two cases:

1. If A is in the last place, then B can be in second, third, or fourth place. Thus, there are 3 possible positions for B , and the remaining three students have ${}_3P_3 = 3! = 6$ possible arrangements. In this case, there are

$$3 \times 6 = 18$$

possible rankings.

2. If A is not in the last place, then A and B must be placed in the second, third, and fourth positions. There are ${}_3P_2 = 6$ possible arrangements for A and B , and the remaining three students also have ${}_3P_3 = 6$ possible arrangements. In this case, there are

$$6 \times 6 = 36$$

possible rankings.

Therefore, in total there are

$$36 + 18 = 54$$

different possible rankings.

Hence, the answer is 54.

4 From 1, 2, 3, 4, 5, and 6, select any three different digits to form a three-digit number. Among all such numbers, how many odd numbers that greater than 300 can be formed?

A. 32

B. 40

C. 56

D. 60

E. 68

Answer B

Solution

Two cases:

If the hundreds digit is **3** or **5**, then there are

$$2 \times 2 \times 4 = 16$$

odd three-digit numbers.

If the hundreds digit is **4** or **6**, then there are

$$2 \times 3 \times 4 = 24$$

odd three-digit numbers.

Therefore, the total number of odd three-digit numbers satisfying the condition is

$$16 + 24 = 40.$$

5 There are **16** players in a chess qualifying tournament. Each pair of players plays exactly one game. In each game, the winner gets **1** point and the loser gets **0** points; if the game is a draw, each player gets **0.5** points. After all the games are finished, players with at least **10** points can advance. What is the maximum possible number of players who can advance in this tournament?

- A. 8 B. 9 C. 10 D. 11 E. 12

Answer D

Solution Sixteen players play a total of $\frac{16 \times 15}{2} = 120$ games, and the total number of points is **120**.

Since a player must score at least **10** points to qualify, the number of qualifiers cannot exceed $\frac{120}{10} = 12$.

First, we prove that having **12** qualifiers is impossible. Suppose there are **12** qualifiers; then there would be **4** non-qualifiers. The **4** non-qualifiers play **6** games among themselves, earning a total of **6** points. Thus, the **12** qualifiers can earn at most **114** points in total. Since the sum of the **12** players' points is $114 < 120$, at least one of them must have fewer than **10** points, contradicting the assumption that all **12** are qualifiers. Hence, there can be at most

11 qualifiers.

Next, we show that having 11 qualifiers is possible. Among the 11 players, there are $\frac{11 \times 10}{2} = 55$ games, totaling 55 points. If all these 55 games end in draws, each player gets 5 points. In addition, each of these 11 players must play against the other 5 non-qualifiers; if each of these games is won, then each player earns another 5 points.

Therefore, each of the 11 players has $5 + 5 = 10$ points, meeting the qualifying standard.

Thus, the answer is 11.

2025 Sept AMC 10 Week 1 Day 2 - Permutation

6 The number of six-digit numbers formed from the digits 0, 1, 2, 3, 4, 5 in which odd and even digits alternate and no digit is repeated is () .

A. 72

B. 60

C. 48

D. 36

E. 12

Answer B

Solution The total number of alternating arrangements of odd and even digits using these six digits is $2({}_3P_3)^2 = 72$.

Among them, the arrangements with 0 in the first place are ${}_3P_3 \times {}_2P_2 = 12$.

Therefore, the number of six-digit numbers satisfying the condition is $72 - 12 = 60$.

Thus, the correct answer is B.

7 Five people A, B, C and two others stand in a row, with A not at either end, and exactly two people standing between B and C . How many different arrangements are there?

A. 20

B. 16

C. 12

D. 8

E. 6

Answer B

Solution Since there are exactly 2 people between B and C , the block consisting of B, C and the two people between them must occupy either the first four positions or the last four positions.

1. When the block occupies the first four positions, the last position remains free. In this case, A must be placed between B and C .

Arranging B and C : ${}_2P_2$ ways.

Placing A between them: ${}_2P_1$ way.

Arranging the other two people in the remaining two positions: ${}_2P_2$ ways.

Thus, the number of arrangements is

$${}_2P_2 \times {}_2P_1 \times {}_2P_2 = 8.$$

2. When the block occupies the last four positions, the first position remains free. Similarly, A must be placed between B and C .

Arranging B and C : ${}_2P_2$ ways.

Placing A between them: ${}_2P_1$ way.

Arranging the other two people in the remaining two positions: ${}_2P_2$ ways.

Thus, the number of arrangements is

$${}_2P_2 \times {}_2P_1 \times {}_2P_2 = 8.$$

By the Addition Principle of Counting, the total number of arrangements is

$$8 + 8 = 16.$$

Therefore, the answer is **B**.

8 If the sum of the digits of a five-digit number is **3**, then the total number of such five-digit numbers is () .

A. 10

B. 12

C. 15

D. 18

E. 20

Answer C

Solution If the sum of the digits of a five-digit number is **3**, then such numbers can be divided into three cases:

Case 1. The digits consist of four **0**'s and one **3**.

Since the first digit cannot be **0**, the **3** must be in the first place and all other digits are **0**, giving the number **30000**.

Thus, there is only **1** possibility.

Case 2. The digits consist of three **0**'s, one **1**, and one **2**.

Since the first digit cannot be **0**, the first digit must be either **1** or **2**. Choose one of them for the first place, and the other digit can occupy any of the remaining four positions. The rest are **0**.

Thus, there are ${}_2C_1 \times {}_4P_1 = 8$ possibilities.

Case 3. The digits consist of two **0**'s and three **1**'s.

Since the first digit cannot be **0**, the first digit must be **1**. Among the remaining four places, choose two for **1**'s, and the rest are **0**.

Thus, there are ${}_4C_2 = 6$ possibilities.

By the Addition Principle of Counting, the total number of such five-digit numbers is

$$1 + 8 + 6 = 15.$$

Therefore, the answer is C.

- 9 There are 5 cars to be parked in 6 adjacent parking spaces. Truck A is wider and occupies two spaces when parked, and car B cannot be parked next to truck A. How many different parking arrangements are there?

A. 72 B. 144 C. 108 D. 96 E. 108

Answer A

Solution First, park truck A.

If truck A is not at the edge, there are 3 possible positions. In this case, car B has 2 possible positions. The other three cars (excluding A and B) can be arranged in ${}_3P_3$ ways.

If truck A is at the edge, there are 2 possible positions. In this case, car B has 3 possible positions. The other three cars can also be arranged in ${}_3P_3$ ways.

Therefore, the total number of parking arrangements is

$$3 \times 2 \times {}_3P_3 + 2 \times 3 \times {}_3P_3 = 36 + 36 = 72.$$

Thus, the answer is A.

- 10 Sammy uses a permutation of the six digits 1, 4, 0, 3, 2, 2 as his six-digit bank card password. If there is exactly one digit between the two 2's, and 1 and 4 are adjacent, then the number of possible passwords is () .

A. 48 B. 32 C. 24 D. 16 E. 8

Answer C

Solution The digits 1 and 4 must be adjacent, which gives ${}_2P_2 = 2$ arrangements.

The two 2's must have exactly one digit between them, which gives ${}_2P_1 = 2$ arrangements.

After grouping these conditions together, the combined blocks can be permuted in ${}_3P_3 = 6$ ways.

Therefore, the total number of possible passwords is $2 \times 2 \times 6 = 24$.

Thus, the answer is C.

2025 Sept AMC 10 Week 1 Day 3 - Combination

11 How many non-negative integer solutions does the equation $x + y + z + t = 6$ have?

A. 72

B. 76

C. 84

D. 90

E. 92

Answer C

Solution This problem is equivalent to placing 6 identical balls into 4 distinct boxes, allowing some boxes to be empty.

Arrange the 6 balls in a row and use 3 dividers to separate them into 4 groups. Since empty boxes are allowed, dividers may be adjacent. Assign the first, second, third, and fourth groups to the variables x, y, z, t respectively.

Because the balls are identical and the dividers are also identical, this is equivalent to arranging 6 balls and 3 dividers in a line. The number of such arrangements is $\binom{9}{3} = 84$.

Therefore, the answer is 84.

12 If three vertices are chosen at random from a regular decagon, the probability that the chosen three points form a right triangle is () .

A. $\frac{1}{4}$

B. $\frac{1}{3}$

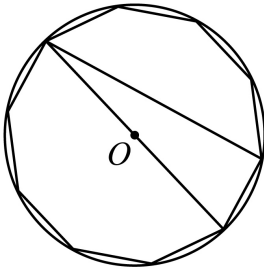
C. $\frac{1}{2}$

D. $\frac{2}{3}$

E. $\frac{3}{4}$

Answer B

Solution



From the 10 vertices of a regular decagon, choosing any 3 points gives ${}_{10}C_3 = 120$ possible selections.

Consider the regular decagon as inscribed in a circle. For the 3 chosen points to form a right triangle, the side opposite the right angle must be a diameter.

There are ${}_5C_1 = 5$ ways to choose a diameter of the circle, and for each diameter, there are ${}_8C_1 = 8$ choices for the right-angle vertex.

Thus, the probability that the chosen 3 points form a right triangle is

$$\frac{{}_5C_1 \times {}_8C_1}{{}_{10}C_3} = \frac{5 \times 8}{120} = \frac{1}{3}.$$

Therefore, the answer is **B**.

13 Four volunteers **A**, **B**, **C**, and **D** are assigned to three soccer fields for volunteer service. Each volunteer must be assigned, and each soccer field must have at least one volunteer. However,

A and B cannot be assigned to the same soccer field. How many different assignment plans are there?

- A. 24 B. 28 C. 30 D. 36 E. 42

Answer C

Solution Since A and B cannot be assigned to the same soccer field, there are two possible cases:

When A and B each serve on different soccer fields, there are ${}_3P_3 = 6$ assignment plans.

When either A or B is assigned together with one of C or D to the same soccer field, there are ${}_2C_1 \times {}_2C_1 \times {}_3P_3 = 24$ assignment plans.

Therefore, the total number of different assignment plans is $6 + 24 = 30$.

Thus, the answer is C.

- 14 A science and technology expo is held in a certain place with 3 exhibition halls. There are 24 volunteer slots to be assigned to these 3 halls, with the requirements that each hall receives at least one slot and the numbers of slots assigned to the halls are all different. How many different assignment methods are there?

- A. 222 B. 244 C. 253 D. 276 E. 284

Answer A

Solution “The distribution method in which each hall receives at least one slot” is equivalent to choosing two gaps from the 23 gaps between the 24 slots to insert dividers, which gives ${}_{23}C_2 = 253$ methods.

The distributions in which at least two halls receive the same number of slots are

$(1, 1, 22), (2, 2, 20), (3, 3, 18), (4, 4, 16), (5, 5, 14), (6, 6, 12), (7, 7, 10), (8, 8, 8), (9, 9, 6), (10, 10, 4), (11, 11, 2),$

for a total of

$10 \times {}_3C_1 + 1 = 31$ cases.

Therefore, the number of distributions in which each hall receives at least one slot and all halls receive different numbers of slots is

$$253 - 31 = 222.$$

Thus, the answer is **A**.

15 Assign 6 doctors to units **A**, **B**, and **C** for nucleic acid testing, with 2 doctors assigned to each unit. Doctor **a** cannot go to unit **A**, and doctor **b** can only go to unit **B**. How many different assignment methods are there?

A. 18

B. 24

C. 30

D. 36

E. 42

Answer A

Solution According to the problem, we discuss two cases:

① If **a** goes to unit **B**, then **a** and **b** are together in unit **B**. The remaining 4 people are divided into 2 groups and assigned to units **A** and **C**. The number of arrangements is $\frac{1}{2} {}_4C_2 \times {}_2P_2 = 6$.

② If **a** does not go to unit **B**, then **a** must go to unit **C**. From the remaining 4 people, choose 2 to be assigned to unit **A**, and then assign the remaining 2 people to units **B** and **C**. The number of arrangements is

$${}_4C_2 \times {}_2P_2 = 12.$$

Therefore, the total number of arrangements is

$$6 + 12 = 18.$$

Thus, the answer is **A**.

2025 Sept AMC 10 Week 2 Day 1 - Classical Probability

16 If one number is chosen at random from all the positive divisors of 2025, what is the probability that this number is a perfect square?

- A. 0.4 B. 0.3 C. 0.6 D. 0.7 E. 0.8

Answer A

Solution Since $2025 = 3^4 \times 5^2$,

the positive divisors of 2025 are

1, 3, 5, 9, 15, 25, 27, 45, 75, 81, 135, 225, 405, 675, 2025,

a total of 15 numbers.

Among them, the perfect squares are

1, 9, 25, 81, 225, 2025,

a total of 6 numbers.

Therefore, if one number is chosen at random from all the positive divisors of 2025, the probability that it is a perfect square is

$$\frac{6}{15} = \frac{2}{5}.$$

Hence, the answer is $\frac{2}{5}$.

17 Assign volunteers A, B, C to classrooms numbered 1, 2, 3 for cleaning, with exactly one volunteer in each classroom. What is the probability that A is not assigned to classroom 3?

- A. $\frac{2}{3}$ B. $\frac{3}{4}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. $\frac{3}{5}$

Answer A

Solution Volunteers A, B, C are assigned to classrooms numbered $1, 2, 3$,

with exactly one volunteer in each classroom. The possible assignments are:

$(A, 1), (B, 2), (C, 3),$

$(A, 1), (B, 3), (C, 2),$

$(A, 2), (B, 1), (C, 3),$

$(A, 2), (B, 3), (C, 1),$

$(A, 3), (B, 1), (C, 2),$

$(A, 3), (B, 2), (C, 1),$

a total of 6.

Among these, the cases where A is not assigned to classroom 3 are:

$(A, 1), (B, 2), (C, 3),$

$(A, 1), (B, 3), (C, 2),$

$(A, 2), (B, 1), (C, 3),$

$(A, 2), (B, 3), (C, 1),$

a total of 4.

Therefore, the probability that A is not assigned to classroom 3 is

$$P = \frac{4}{6} = \frac{2}{3}.$$

Thus, the answer is A .

18 From five line segments of lengths $2, 4, 6, 8, 10$, three are chosen at random. The probability that these three line segments can form a triangle is () .

A. $\frac{1}{5}$

B. $\frac{3}{10}$

C. $\frac{2}{5}$

D. $\frac{3}{5}$

E. $\frac{7}{10}$

Answer B

Solution Choosing any 3 line segments, the possible sets are

$\{2, 4, 6\}, \{2, 4, 8\}, \{2, 4, 10\}, \{2, 6, 8\}, \{2, 6, 10\}, \{2, 8, 10\}, \{4, 6, 8\}, \{4, 6, 10\}, \{4, 8, 10\},$ a
 $\{6, 8, 10\},$

total of 10 basic events.

Among these, the sets of three line segments that can form a triangle are

$\{4, 6, 8\}$, $\{4, 8, 10\}$, $\{6, 8, 10\}$, a total of 3.

Therefore, the probability that three chosen line segments can form a triangle is

$$P = \frac{3}{10}.$$

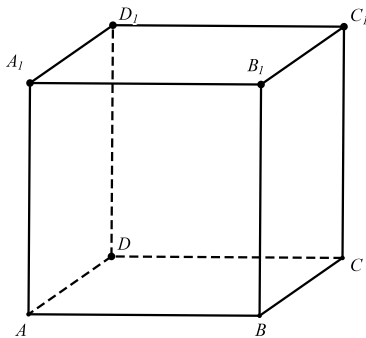
Thus, the answer is **B**.

19 If three vertices are chosen at random from the eight vertices of a cube and connected to form a triangle, what is the probability that the triangle is equilateral?

- A. $\frac{1}{7}$ B. $\frac{1}{14}$ C. $\frac{2}{7}$ D. $\frac{4}{35}$ E. $\frac{5}{14}$

Answer A

Solution The number of ways to choose any three vertices from the eight vertices to form a triangle is ${}_8C_3 = 56$.



Among them, there are 8 outcomes that form an equilateral triangle:

$\triangle ACD_1$, $\triangle BDC_1$, $\triangle ACB_1$, $\triangle BDA_1$, $\triangle A_1C_1B$, $\triangle B_1D_1A$, $\triangle B_1D_1C$, $\triangle A_1C_1D$.

Therefore, the probability is $\frac{8}{56} = \frac{1}{7}$, hence the answer is **A**.

To assess students' physical fitness, 3 items are chosen at random from a total of 7 events: 3 track and field events, 2 ball games, and 2 martial arts events. What is the probability that the chosen 3 items come from exactly two categories?

- A. $\frac{22}{35}$ B. $\frac{2}{7}$ C. $\frac{4}{7}$ D. $\frac{11}{35}$ E. $\frac{33}{35}$

Answer A

Solution The total number of ways to choose 3 items from these 7 events is

$${}_7C_3 = 35.$$

The number of ways where all 3 items come from the same category is

$${}_3C_3 = 1.$$

The number of ways where the 3 items cover all three categories is

$${}_3C_1 {}_2C_1 {}_2C_1 = 12.$$

Therefore, the number of favorable cases (exactly two categories) is

$$35 - 1 - 12 = 22.$$

So the required probability is $\frac{22}{35}$.

Hence, the answer is $\frac{22}{35}$.

2025 Sept AMC 10 Week 2 Day 2 - Geometric Probability

21 A point P is chosen at random inside a circle C with radius 2. What is the probability that the chord with midpoint P has length less than $2\sqrt{3}$?

- A. $\frac{1}{2}$ B. $\frac{3}{4}$ C. $\frac{1}{4}$ D. $\frac{3}{10}$ E. $\frac{3}{5}$

Answer B

Solution From the problem we have:

The chord with midpoint P has length less than $2\sqrt{3}$ if and only if the distance from the circle's center to P satisfies

$$d > \sqrt{2^2 - \left(\frac{2\sqrt{3}}{2}\right)^2} = 1.$$

That is, when $|CP| > 1$, the chord with midpoint P has length less than $2\sqrt{3}$.

By the geometric probability formula, the required probability is

$$P = \frac{\pi \cdot 2^2 - \pi \cdot 1^2}{\pi \cdot 2^2} = \frac{3}{4}.$$

Therefore, the answer is $\frac{3}{4}$.

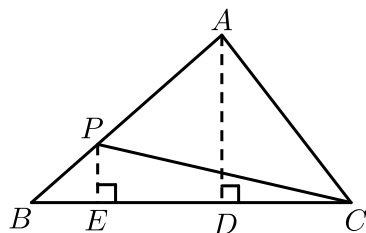
22 On side AB of triangle ABC with area S , a point P is chosen at random. What is the probability that the area of $\triangle PBC$ is greater than $\frac{S}{3}$?

- A. $\frac{1}{6}$ B. $\frac{2}{3}$ C. $\frac{1}{4}$ D. $\frac{1}{3}$ E. $\frac{5}{6}$

Answer B

Solution Let the event be $A = \left\{ \text{the area of } \triangle PBC \text{ is greater than } \frac{S}{3} \right\}$.

The sample space is the length of segment AB (see figure).



Since $S_{\triangle PBC} > \frac{S}{3}$,

we have $\frac{1}{2} \cdot BC \cdot PE > \frac{1}{3} \times \frac{1}{2} \cdot BC \cdot AD$,

which simplifies to $\frac{PE}{AD} > \frac{1}{3}$.

Since $PE \parallel AD$, by the similarity $\triangle BPE \sim \triangle BAD$, we obtain $\frac{BP}{AB} = \frac{PE}{AD} > \frac{1}{3}$.

Thus, the geometric measure of event A corresponds to the length on segment AB .

Since $AP = \frac{2}{3}AB$, we get $P(A) = \frac{AP}{AB} = \frac{2}{3}$.

Therefore, the answer is **B**.

23 As shown in the figure, circles M , N , and P are mutually externally tangent and also tangent internally to the equilateral triangle ABC . If a point is chosen at random inside equilateral triangle ABC , what is the probability that this point lies inside triangle MNP (the shaded region)?

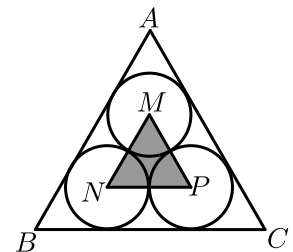
A. $\frac{\sqrt{3}-1}{2}$

B. $\frac{\sqrt{3}-1}{3}$

C. $\frac{2-\sqrt{3}}{2}$

D. $\frac{2-\sqrt{3}}{3}$

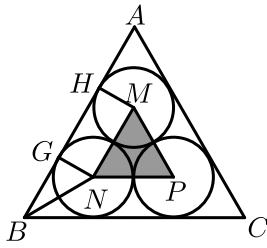
E. $\frac{\sqrt{2}-1}{2}$



Answer C

Solution As shown in the figure, let the radius of one inscribed circle be r . Then

$$AH = BG = \sqrt{3}r,$$



so $MN = GH = 2r$, and $AB = AH + BG + GH = 2(\sqrt{3} + 1)r$.

Since equilateral triangle MNP is similar to equilateral triangle ABC ,

the probability that a randomly chosen point inside ABC lies in triangle MNP (the shaded region) is

$$P = \frac{A_{\triangle MNP}}{A_{\triangle ABC}} = \left(\frac{MN}{AB} \right)^2 = \left(\frac{2r}{2(\sqrt{3} + 1)r} \right)^2 = \frac{2 - \sqrt{3}}{2}.$$

Therefore, the answer is C.

- 24 Since Wong is busy at work and has no time to cook lunch, he orders takeout. Suppose both Wong and the delivery person both arrive randomly between 12:00 and 12:10 at the entrance of Wong's building. Suppose Wong arrived earlier than the delivery person, what is the probability that Wong waits no more than 5 minutes for the delivery person?

- A. $\frac{1}{2}$ B. $\frac{4}{5}$ C. $\frac{3}{4}$ D. $\frac{3}{8}$ E. $\frac{5}{8}$

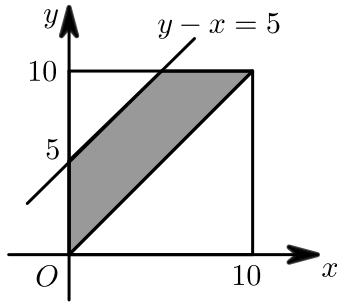
Answer D

Solution Let the arrival times (in minutes after 12 : 00) of the delivery person and Wong be x and y respectively. The event that "Wong waits no more than 5 minutes" is event A .

The sample space is $\Omega = \{(x, y) \mid 0 \leq x \leq 10, 0 \leq y \leq 10\}$, and

$$A = \{(x, y) \mid 0 \leq x \leq 10, 0 \leq y \leq 10, 0 \leq y - x \leq 5\}.$$

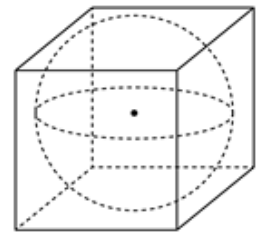
(see figure)



$$P(A) = \frac{\frac{1}{2}(10 \times 10 - 5 \times 5)}{10 \times 10} = \frac{3}{8}.$$

Choose D.

- 25 As shown in the figure, a sphere is inscribed in a cube of side length 2 (the sphere is tangent to each face of the cube). If a point P is chosen at random inside the cube, what is the probability that the point lies outside the sphere?



- A. $\frac{\pi}{6}$ B. $\frac{\pi}{8}$ C. $\frac{6-\pi}{6}$ D. $\frac{8-\pi}{8}$ E. $\frac{\pi}{4}$

Answer C

Solution $V_{\text{cube}} = 2 \times 2 \times 2 = 8$.

The radius of the inscribed sphere is 1, so its volume is $V = \frac{4}{3}\pi \times 1^3 = \frac{4}{3}\pi$.

The volume of the cube is $2^3 = 8$.

Therefore, the probability that a randomly chosen point in the cube lies outside the sphere is

$$\frac{8 - \frac{4}{3}\pi}{8} = \frac{6 - \pi}{6}.$$

Thus, the answer is C.

- 26 A student's shooting accuracy for one shot is $\frac{9}{10}$, and the probability of making two consecutive shots is $\frac{1}{2}$. If the student has just made a shot, then what is the probability that the next shot is also successful?

A. $\frac{1}{5}$ B. $\frac{2}{5}$ C. $\frac{3}{5}$ D. $\frac{5}{9}$ E. $\frac{2}{9}$

Answer D

Solution If A_i ($i = 1, 2$) denotes the event that the i -th shot is successful, then

$$P(A_1) = \frac{9}{10}, P(A_1 A_2) = \frac{1}{2}.$$

$$\text{Therefore, } P(A_2 | A_1) = \frac{P(A_1 A_2)}{P(A_1)} = \frac{5}{9}.$$

Thus, the answer is D.

- 27 A and B each shoot once at the same target. The probability that A hits the target is $\frac{1}{3}$, and the probability that B hits the target is $\frac{1}{2}$. Given that the target is hit at least once, what is the probability that A hits the target?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$ E. $\frac{3}{4}$

Answer C

Solution Let event A be: the target is hit at least once, and event B be: A hits the target.

$$\text{We calculate: } P(A) = \frac{1}{3} \times \frac{1}{2} + \left(1 - \frac{1}{3}\right) \times \frac{1}{2} + \frac{1}{3} \times \left(1 - \frac{1}{2}\right) = \frac{2}{3},$$

$$P(AB) = \frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \left(1 - \frac{1}{2}\right) = \frac{1}{3}.$$

$$\text{So, } P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}.$$

Therefore, the answer is C.

- 28 A medical device has two consumable components, A and B . After each use, the probability that component A needs to be replaced is 0.3, and the probability that component B needs to be replaced is 0.5. Given that at least one component must be replaced after the first use, what

is the probability that both A and B need to be replaced?

- A. 0.15 B. 0.65 C. $\frac{3}{13}$ D. $\frac{5}{13}$ E. $\frac{6}{13}$

Answer C

Solution Let event E be: at least one component needs to be replaced after the first use, and event F be: both components A and B need to be replaced.

Then,

$$P(E) = 1 - (1 - 0.3)(1 - 0.5) = 0.65,$$

$$P(EF) = 0.3 \times 0.5 = 0.15.$$

By the conditional probability formula,

$$P(F | E) = \frac{P(EF)}{P(E)} = \frac{0.15}{0.65} = \frac{3}{13}.$$

Therefore, the answer is C .

29 Two players A and B compete for the championship in a Go match. The match follows a “best of three” format. The probability that A wins any individual game is $\frac{2}{3}$, and the outcomes of different games are independent. Given that A wins the championship, what is the probability that the match lasts for 3 games?

- A. $\frac{3}{10}$ B. $\frac{7}{10}$ C. $\frac{1}{2}$ D. $\frac{2}{5}$ E. $\frac{1}{5}$

Answer D

Solution From the problem, the probability that A wins the championship is

$$\frac{2}{3} \cdot \frac{2}{3} + \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{20}{27}.$$

$$\text{The probability that the match lasts for 3 games is } \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}.$$

Therefore, given that A wins the championship, the probability that the match lasted for 3

$$\text{games is } \frac{\frac{8}{27}}{\frac{20}{27}} = \frac{2}{5}.$$

Hence, the answer is $\frac{2}{5}$.

30 Andy can choose to join from 2 science clubs, 4 art clubs, and 3 sports clubs. If Andy signs up for two clubs, then given that exactly one of them is an art club, what is the probability that the other one is a sports club?

A. $\frac{3}{5}$

B. $\frac{6}{13}$

C. $\frac{1}{2}$

D. $\frac{3}{4}$

E. $\frac{2}{5}$

Answer A

Solution Let event A be “one is an art club,” and event B be “one is a sports club,”

then

$$P(A) = \frac{{}^4C_1 \times {}^5C_1}{{}^9C_2} = \frac{20}{36} = \frac{5}{9},$$

$$P(AB) = \frac{{}^4C_1 \times {}^3C_1}{{}^9C_2} = \frac{12}{36} = \frac{1}{3}.$$

$$\text{Thus, } P(B|A) = \frac{P(AB)}{P(A)} = \frac{\frac{1}{3}}{\frac{5}{9}} = \frac{3}{5}.$$

Therefore, the answer is **A**.