2025 Sept AMC 8 Week 1&2

2025 Sept AMC 8 Week 1 Day 1 - Counting Principles

1. From City A to City B, one can travel by bus, ship, or train. The bus runs 3 times a day, the ship 2 times a day, and the train 6 times a day. In total, how many different ways are there that travel from City A to City B in one day?

A. 7 B. 9 C. 11 D. 13 E. 15

Answer C

Solution Since one can travel directly from City A to City B by bus, ship, or train, this problem can be considered in three cases.

Taking the bus gives 3 ways, taking the ship gives 2 ways, and taking the train gives 6 ways.

Therefore, the total number of ways to travel from City A to City B is

$3+2+6=11 \quad (\text{ways})$.

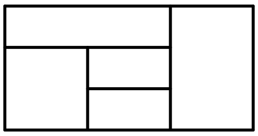
2. Four students form a study group. A leader and a deputy leader are to be chosen from among the four. In total, there are _____ different ways to make this selection.

A. 6 B. 8 C. 10 D. 12 E. 14

Answer D

Solution $4\times3=12$

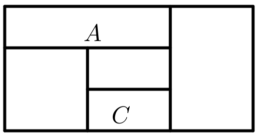
3. 4 colors are available (not necessary to use all) to paint each part of the figure below. If the colors in adjacent parts cannot be the same, how many ways of painting are there?



A. 144 B. 288 C. 72 D. 36 E. 6

Answer C

Solution Case analysis:



First fill the middle part, which has 4 possibilities.

(1) AC has the same color:

$4 \times 3 \times 1 \times 2 \times 2 = 48$

(2) AC has different colors:

$4 \times 3 \times 2 \times 1 \times 1 = 24$

Thus,

$48 + 24 = 72$ ways.

Therefore, the answer is: C.

4 In a certain school's ballroom dance troupe, there are 43 members in total. Among them, 15 can dance Latin, 13 can dance Tango, and 5 can dance both. The number of people who can dance neither of these two dances is ______ .

A. 18 B. 20 C. 22 D. 24 E. 26

Answer B

Solution $43 - (15 + 13 - 5) = 20$

5

Among 40 students solving three math problems, 25 solved the first problem correctly, 28 solved the second problem correctly, and 31 solved the third problem correctly. How many students solved all three problems correctly, at least.

A. 3 B. 4 C. 5 D. 6 E. 7

Answer B

Solution The number of students who solved both the second and the third problems is

$28+31-40=19 \quad (\text{students}),$

and the number of students who solved all three problems is

$40-(25+28+31-13-16-19)=4 \quad (\text{students}).$

2025 Sept AMC 8 Week 1 Day 2 - Permutation

6 There are 9 different books: 3 math books, 4 Chinese books, and 2 English books. These books are to be arranged in a row on a shelf, with the math books kept together and the Chinese books kept together. Then there are ______ possible arrangements in total.

Answer 3456

Solution Since the books of the same subject must be placed together, we can “bundle” them. First, bundle the math books and the Chinese books, making two bundles. Together with the two English books, we have four items to arrange. After arranging these, we then order the books within each bundle.

Therefore, the total number of arrangements is ${}_4P_4 \times {}_4P_4 \times {}_3P_3 = 3456 \quad (\text{ways}).$

7 Five people line up for a photo. Person A does not want to stand at either end. How many different possible arrangements are there?

A. 72 B. 48 C. 36 D. 24 E. 12

Answer A

Solution $3 \times 4 \times 3 \times 2 \times 1 = 72$

8 It's primary school graduation, and June's group is taking a photo. There are 4 boys and two girls, June and Ming. The two girls must not stand at either end and must stand next to each other. How many different possible arrangements are there?

A. 24 B. 48 C. 96 D. 144 E. 184

Answer D

Solution ${}_4P_4 \times 2 \times {}_3C_1$

$= 24 \times 6$

$= 144$

9 There are 3 boys and 2 girls standing in a line. The two girls are not allowed to stand next to each other. Then there are _____ different possible arrangements.

Answer 72

Solution For 5 people in a line, normally there are ${}_5P_5 = 120$ arrangements.

If the two girls are adjacent, there are $2 \times {}_4P_4 = 48$ arrangements.

Therefore, the total number of arrangements is $120 - 48 = 72$.

10 Class 3 of Grade 4 is holding a Children's Day celebration. The entire program consists of 2 dances, 2 songs, and 3 skits. If programs of the same type must be performed consecutively, then there are _____ different possible performance orders.

A. 122 B. 144 C. 155 D. 166 E. 177

Answer B

Solution Since programs of the same type must be performed consecutively, we can apply the "bundling method." First, arrange the three types of programs (dance, song, skit). Then, within each type, arrange the specific programs.

Therefore, the total number of performance orders is

${}_3P_3 \times {}_2P_2 \times {}_2P_2 \times {}_3P_3 = 144 \quad \text{(orders)}.$

2025 Sept AMC 8 Week 1 Day 3 - Combination

11 Pick 3 from 8 children to take part in interview. How many different combination(s) is / are there ?

A. 36 B. 42 C. 48 D. 56 E. 72

Answer D

Solution $8 \times 7 \times 6 \div 6 = 56$.

12 3 of the 5 children are selected to participate in a competition, and Chris and Debbie are 2 of the 5 children. If at least one people between Chris and Dibbie is selected, how many ways of selecting participants are there?

A. 17 B. 2021 C. 9 D. 5 E. 1

Answer C

Solution From 5 children, choosing 3 can be done in ${}_5C_5 = \dfrac{5 \times 4 \times 3}{3 \times 2 \times 1} = 10$ ways.

The case where neither Chris nor Debbie is chosen means choosing all 3 from the remaining 3 children, which is ${}_3C_3 = 1$ way.

Therefore, the number of ways in which at least one of the two is chosen is $10-1=9$.

13 Brandon needs to choose three courses to study from Physics, Chemistry, Biology, Politics, History, and Geography. He has already chosen Physics. How many different ways are there for him to choose the other two courses?

A. 10 B. 15 C. 20 D. 30

Answer A

Solution There are ${}_5C_2=10$ ways.

14 Connect the diagonals of square $ABCD$, and color each of the four vertices either red or yellow. A triangle whose vertices are all the same color is called a monochromatic triangle. How many coloring methods in which there is at least one monochromatic triangle?

A. 12 B. 17 C. 15 D. 22 E. 10

Answer E

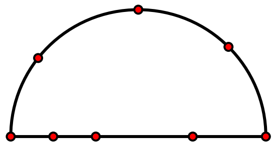
Solution Each vertex can be colored in two ways, so there are $2\times2\times2\times2=16$ coloring methods in total.

For there to be a monochromatic triangle, the case of “two vertices red and two vertices yellow” must be excluded. This case has ${}_4C_2=6$ methods. Therefore, the number of coloring methods that yield at least one monochromatic triangle is $16-6=10$.

Equivalently, to have a monochromatic triangle, we must exclude the case where “the two diagonals are colored differently.” This case has $2\times3=6$ methods. Hence, the number of coloring methods with at least one monochromatic triangle is also $16-6=10$.

15

From the 8 points shown in the figure, choosing any 3 as vertices, how many triangles can be formed?



A. 40 B. 42 C. 44 D. 46 E. 52

Answer D

Solution ${}_8C_3 - {}_5C_3 = 46$

2025 Sept AMC 8 Week 2 Day 1 - Classical Probability

16 There are three cards with numbers 8, 6, and 10. If any two cards are drawn, the difference that is most likely to occur is ______ .

Answer 2

Solution If 8 and 6 are drawn, their difference is $8-6=2$

If 8 and 10 are drawn, their difference is $10-8=2$

If 6 and 10 are drawn, their difference is $10-6=4$

Since the difference of 2 occurs more frequently, the most likely difference is 2.

17 There are two dice, one large and one small. Each die has six faces numbered from 1 to 6. When the two dice are thrown simultaneously, what is the probability that the product of the two numbers is 12?

A. $\frac{5}{36}$ B. $\frac{1}{9}$ C. $\frac{1}{18}$ D. $\frac{5}{18}$ E. $\frac{2}{9}$

Answer B

Solution List all possibilities:

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

The probability that the product of the two numbers is 12 is $\frac{4}{36}=\frac{1}{9}$.

Therefore, the answer is $\frac{1}{9}$.

18 In the sixth grade of a certain primary school, there are 6 classes, each with 40 students. Two classes are randomly selected from the 6 classes to participate in a live entertainment event hosted by a TV station. During the event, there is one lottery in which 4 lucky audience members are chosen. What is the probability that Bunny, a sixth-grade student, becomes one of the lucky winners?

A. $\frac{1}{60}$ B. $\frac{1}{20}$ C. $\frac{1}{30}$ D. $\frac{1}{15}$ E. $\frac{1}{10}$

Answer A

Solution The probability that Bunny's class is selected to participate in the event is $\frac{{}_5C_1}{{}_6C_2}=\frac{5}{15}=\frac{1}{3}$.

If Bunny participates in the event, then the probability that he becomes a lucky winner is $\frac{4}{40\times 2}=\frac{1}{20}$.

Therefore, the probability that Bunny becomes a lucky winner is $\frac{1}{3}\times\frac{1}{20}=\frac{1}{60}$.

Alternatively, the probability that Bunny's class is selected can also be written as $\frac{5}{6\times 5\div 2}=\frac{1}{3}$.

Then, combining with the lottery probability, we again obtain $\frac{1}{3}\times\frac{1}{20}=\frac{1}{60}$.

19 A school offers 3 labor-technology courses and 4 art courses. A student selects 3 courses from them. The probability that the student selects at least one course from each category is ______ .

A. $\frac{6}{7}$ B. $\frac{5}{7}$ C. $\frac{4}{7}$ D. $\frac{3}{7}$ E. $\frac{1}{7}$

Answer A

Solution $\frac{{}_7C_3 - {}_3C_3 - {}_4C_3}{{}_7C_3} = \frac{6}{7}$.

20 Weier is playing a coin-tossing game: If a coin is tossed 5 times, what is the probability of getting heads no more than twice?

A. $\frac{1}{2}$ B. $\frac{1}{8}$ C. $\frac{5}{32}$ D. $\frac{1}{32}$ E. $\frac{1}{4}$

Answer A

Solution The probability of getting 0 heads is $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$, the probability of getting 1 head is $5 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{32}$, the probability of getting 2 heads is ${}_5C_2 \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{5}{16}$.

Therefore, the probability of getting no more than 2 heads is $\frac{1}{32} + \frac{5}{32} + \frac{5}{16} = \frac{1}{2}$.

2025 Sept AMC 8 Week 2 Day 2 - Number Problems

21 Using the digits $1, 2, 3, 4$ to form three-digit numbers without repeating digits, if one such number is chosen at random, what is the probability that the selected number is a multiple of 3?

A. $\frac{1}{4}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{3}{4}$ E. $\frac{1}{3}$

Answer C

Solution There are 4 possible combinations to form three-digit numbers. Among them, the combinations $(1, 2, 3)$ and $(2, 3, 4)$ produce multiples of 3, giving a total of 2 such combinations.

Therefore, the probability is $\frac{2}{4}=\frac{1}{2}$.

22 Given the four digits $2, 0, 1, 7$, each digit may be used at most once. How many natural numbers less than 2017 can be formed?

A. 33 B. 35 C. 37 D. 41 E. 45

Answer C

Solution The natural numbers that satisfy the condition can be divided into 4 categories:

One-digit numbers: $0, 1, 2, 7$, a total of 4.

Two-digit numbers: The first digit cannot be 0, and digits cannot be repeated, so there are $3\times 3=9$ numbers.

Three-digit numbers: The first digit cannot be 0, and digits cannot be repeated, so there are $3\times 3\times 2=18$ numbers.

Four-digit numbers: The first digit cannot be 0, and digits cannot be repeated, so there are $3\times 2\times 1=6$ numbers.

Therefore, in total, the number of natural numbers less than 2017 that can be formed is $4+9+18+6=37$.

23 From 3 ones, 2 twos, and 1 three, if 3 digits are selected, how many different three-digit numbers can be formed?

A. 15 B. 16 C. 17 D. 18 E. 19

Answer E

Solution Case 1:

3 digits all the same: 111, only 1.

Case 2:

2 of three digits are the same:

322, 232, 223, 311, 131, 113, 122, 212, 221, 211, 121, 112. There are $3+3+6=12$ numbers.

Case 3:

All three digits are different: there are ${}_3P_3=3\times2\times1=6$ numbers.

Totally, $1+12+6=19$ numbers.

24 Among the positive integers less than 1000, how many are there with no repeating digit?

A. 648 B. 738 C. 758 D. 828 E. 670

Answer B

Solution Three-digit numbers: 648

Two-digit numbers: 81

One-digit numbers: 9

In total: 738.

25 Using the digits $1,2,3,4,5$ to form a five-digit number (each digit used at most once), such that the difference between any two adjacent digits is at least 2. How many such five-digit numbers are there?

A. 10 B. 12 C. 14 D. 16 E. 18

Answer C

Solution Starting with 1: 2 numbers — $13524, 14253$;

so starting with 5, there are also 2 numbers.

Starting with 2: 3 numbers — $24135, 24153, 25314$;

so starting with 4, there are also 3 numbers.

Starting with 3: 4 numbers — $31425, 31524, 35241, 35142$.

Starting with 4: 3 numbers — $41352, 42513, 42531$.

Starting with 5: 2 numbers — $52413, 53142$.

Therefore, the total number of such five-digit numbers is 14.

2025 Sept AMC 8 Week 2 Day 3 - Drawing Problems

26 There are 5 red balls and 3 white balls in the box. If two balls are drawn at random, what is the probability of getting one red and one white?

A. $\frac{13}{28}$ B. $\frac{15}{28}$ C. $\frac{15}{56}$ D. $\frac{5}{14}$ E. $\frac{5}{7}$

Answer B

Solution $\frac{5\times 3}{8\times 7\div 2}=\frac{15}{28}$.

27 Box A contains 3 white ping-pong balls numbered $1, 2, 3$. Box B contains 3 yellow ping-pong balls numbered $4, 5, 6$. One ball is randomly drawn from each box. What is probability that the sum of the two numbers is greater than 6?

A. $\frac{1}{4}$ B. $\frac{1}{2}$ C. $\frac{2}{3}$ D. $\frac{3}{4}$ E. $\frac{1}{3}$

Answer C

Solution The possible sums of the numbers are:

$1+4=5,\ 1+5=6,\ 1+6=7,\ 2+4=6,\ 2+5=7,\ 2+6=8,\ 3+4=7,\ 3+5=8,\ 3+6=9.$

The probability that the sum is greater than 6 is $\frac{6}{9}=\frac{2}{3}$.

Therefore, the answer is $\frac{2}{3}$.

28 A bag contains 5 red balls, 6 white balls, and 3 black balls. In order for the probability of drawing a black ball to be $\frac{2}{3}$, how many additional black balls must be put into the bag?

A. 15 B. 16 C. 17 D. 18 E. 19

Answer E

Solution Suppose x additional black balls need to be put into the bag. Then $\frac{3+x}{5+6+3+x}=\frac{2}{3}$.

Solving gives $x=19$.

Therefore, 19 more black balls should be added to the bag.

29 Vendors often run lottery games at the school gate. One vendor has a black bag containing 50 balls of different colors: 1 red, 2 yellow, 10 green, and the rest white. After mixing them thoroughly, the rule is: each draw costs 2 dollars for 1 ball.

Drawing a red ball wins a prize worth 8 dollars.

Drawing a yellow ball wins a prize worth 5 dollars.

Drawing a green ball wins a prize worth 2 dollars.

Drawing a white ball wins no prize.

If you spend 4 dollars to draw 2 balls at the same time, what is the probability of obtaining a prize worth 10 dollars?

A. $\frac{18}{1225}$ B. $\frac{11}{1225}$ C. $\frac{121}{225}$ D. $\frac{11}{245}$ E. $\frac{2}{245}$

Answer B

Solution There are two cases for winning a prize worth 10 dollars:

Case 1: Drawing one 8-dollar prize and one 2-dollar prize, i.e., one red ball and one green ball. The probability of drawing a red ball first and then a green ball, or a green ball first and then a red ball, is the same, giving $\frac{1\times 10}{50\times 49}\times 2=\frac{2}{245}$.

Case 2: Drawing two yellow balls. The probability of drawing the first yellow ball is $\frac{2}{50}$, and the probability of drawing the second yellow ball is $\frac{1}{49}$. Thus, $\frac{2}{50}\times\frac{1}{49}=\frac{1}{1225}$.

Therefore, the total probability is $\frac{2}{245}+\frac{1}{1225}=\frac{11}{1225}$.

30 A bag contains 5 balls that are identical in size and shape: 3 white balls and 2 red balls. Two balls are drawn with replacement. What is the probability that the two balls are of different

colors?

A. $\frac{2}{5}$ B. $\frac{11}{25}$ C. $\frac{12}{25}$ D. $\frac{13}{25}$ E. $\frac{3}{5}$

Answer C

Solution The probability that both balls are white is $\frac{3 \times 3}{5 \times 5}=\frac{9}{25}$, and the probability that both balls are red is $\frac{2 \times 2}{5 \times 5}=\frac{4}{25}$.

So the probability that the two balls are the same color is $\frac{9}{25}+\frac{4}{25}=\frac{13}{25}$.

Therefore, the probability that the two balls are of different colors is $1-\frac{13}{25}=\frac{12}{25}$.