Think Academy US Online

The Ultimate Geometry Guide for Middle Schoolers

2025

Featuring Past Math Contest Questions & In-Depth Explanations

Contents

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SCAN ME

https://www.thethinkacademy.com/

学而思美国ONLINE

中学数学课程体系

Think Academy 中学长期班体系专为美国G6-G9的初中生打造，提供专业、系统且全面的全年数学课程。课程依据北美学生的学习特点与需求，分为Core+体系（校内同步），Honors体系（超前升学），Challenge体系和Competition体系。

Core+体系（校内同步体系）：通过预习并巩固重点知识，精准练习，实现校内同年进度数学轻松拿A的目标。

Honors体系（超前升学体系）：超前中学正常进度1年，匹配公校最快班进度，并加深学习难度，实现进入公校最快班，高中毕业前修完5门理科AP，SAT/ACT数学满分的目标

Challenge体系：超前中学正常进度2年，在Honors班的基础上继续加快进度，实现两年学完Algebra 1，Geometry和Algebra 2的目标，进度和深度匹配顶尖私校，高中毕业前修完8门理科AP，达到SAT/ACT数学满分的目标。

Competition体系：中学AMC竞赛体系，专门针对AMC考纲设计，通过超纲匹配竞赛考点的体系，培养过上百名获奖选手的专业教练团队授课，帮助中学孩子在G8/9前晋级AIME。

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中学竞赛体系学员成绩

2022-2024 AMC8累计获奖学员人数：

456
Achievement Roll
（低年级成就奖）

397
Honor Roll
（全国Top 5%）

134
DHR
（全国Top 1%）

2022-2024 AMC10累计获奖学员人数：

304
AIME晋级
（全国Top 7%）

187
Honor Roll
（全国Top 5%）

45
DHR
（全国Top 1%）

2024 Think全球IMO获奖人数

7金 1银

金牌总数居全球第一

International Mathematical Olympiad
Foundation Bronze Sponsor

Think Academy是国际数学奥林匹克竞赛官方合作伙伴

Think竞赛课程为什么能培养上千位获奖学员?

1 专业竞赛体系，一站式解决竞赛学习

Think Competition根据美国数学竞赛AMC的考纲设计，贴合学生的考试节奏，在**5-6年级学习AMC8**，**7-8年级学习AMC10**，知识点涵盖竞赛的四大模块：代数，数论，数论，和计数概率，从而每年实现一个竞赛目标，最终帮助中学生**在进入高中前完成AMC10的学习，顺利晋级AIME**。

2 优秀竞赛师资，为好成绩保驾护航

Think Competition课程均由**多年竞赛授课经验**的老师授课，让孩子可以更高效且轻松的掌握复杂竞赛知识点。

James老师
宾大-5年竞赛教龄

Dennis老师
杜克大学-4年竞赛教龄

Yichen老师
哥大-4年竞赛教龄

3 和优秀的同龄人一起学习，共同进步

竞赛体系每年的课程均设置入学考试，确保每位学生可以和水平相近的同龄人一起学习，让竞赛备考不再孤单，孩子们可以互相激励，共同进步。

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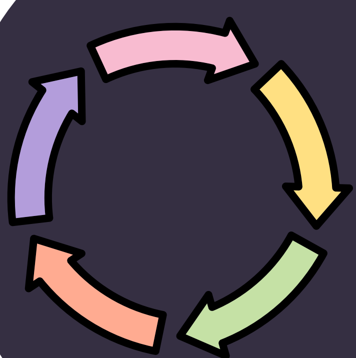
课程亮点

家长省心，规划清晰

授课老师为孩子定制学习规划，全程跟踪学习进度

- **报名课程：**学习规划老师针对孩子的学习能力与目标，**制定个性化学习方案**。
- **上课期间：**每月和家长**反馈孩子的学习情况**，提供有针对性的学习建议，并**监督落实孩子的提升方案**。
- **期中/期末：**每学期组织**家长会**，梳理孩子的学习优势和薄弱环节，并制定新学期的学习规划。



每周学习闭环，保障学习效果

- **课前预习：**15分钟**课前预习题**，温故而知新
- **课后作业：**每节课**配套作业**题目，老师主动和家长反馈学生的作业完成情况。
- **Office Hour：免费作业讲解直播课**，解答孩子课后不明白的题目与知识点。
- **作业解析：**每道作业配套**讲解视频**，随时复习错题

全年学习服务支持

- 专业客服，**全年 364 天 Parent APP 在线支持**，快速响应任何问题
- Parents App**直接和授课老师联系**，沟通更高效，随时掌握孩子的学情表现。
- 在线作业答疑，给孩子**最及时的学习帮助**

Lesson 1 Angles

Concept 1: Classification and Notation of Angles

1. Angle: An **angle** is composed of two different rays with the same endpoint.

2. Classification of Angles:

(1) A **straight angle is** an angle whose measure is **180°**.

(2) A **right angle** is an angle whose measure is **90°**.

(3) An acute angle is an angle whose measure is greater than **0° and less than 90°.**

(4) An obtuse angle is an angle whose measure is greater than 90**° and less than 180°.**

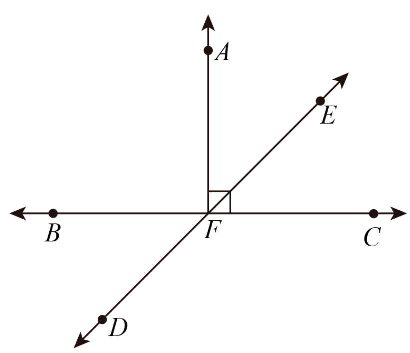
3. Notation of an angle: An angle in geometry is denoted by the **angle symbol (∠) followed by three letters** which represent points that form the angle.

4. Complementary and supplementary angles:
Two angles are **complementary angles** if sum of their measures is 90°; two angles are **supplementary angles** if if sum of their measures is 180°.

5. Vertical Angles and Adjacent angles
Vertical angles: Two opposite angles which are formed by two intersecting lines. They have the same measures.

Adjacent angles: Two angles that share the same vertex and exactly one side. The sum of their measure is 180°.

Linear Pairs: A linear pair is **a pair of adjacent angles formed when two lines intersect**. The sum of their measures is 180.



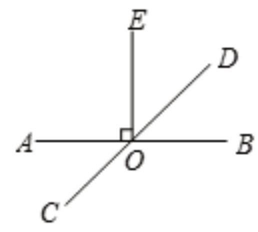
Math Exploration 1

As shown in the figure below, line AB meets line CD at O. $OE \perp AB$ and $\angle BOD = 45^\circ$.

(1) $\angle COE =$ ______ $^\circ$, it is an ______ (obtuse/acute) angle.

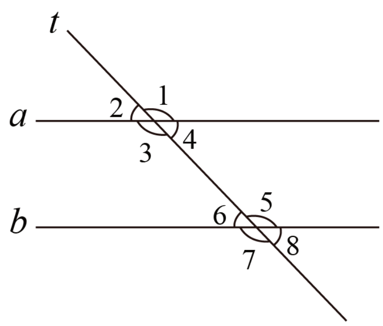
(2) $\angle COA =$ ______ $^\circ$, it is an ______ (obtuse/acute) angle.

(3) $\angle COA$ and $\angle COE$ are ______ (complementary/supplementary) angles.



Concept 2: Parallel Lines

If two lines in a plane do not meet, we say that they are **parallel.** As shown in the figure below, line a is parallel to line b and we can denote it as $a//b$.



When a transversal t intersects two lines a and b:

two angles are **corresponding angles** if they occupy corresponding positions ($\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$);

two angles are **alternate interior angles** if they lie between the two lines on opposite sides of the transversal ($\angle 3$ and $\angle 5$, $\angle 4$ and $\angle 6$);

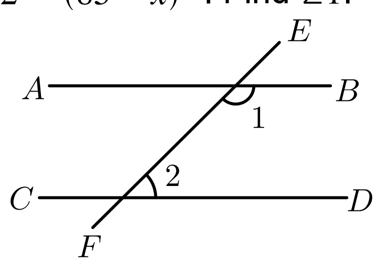
two angles are **same-side interior angles** which lie on the same side of the tranversal and lie between two lines. ($\angle 4$ and $\angle 5$, $\angle 3$ and $\angle 6$).

If a **transversal** t intersects two **parallel lines** a **and** b, then **corresponding angles, alternate interior angles** are congruent and the sum of **same-side interior angles is** 180°**.**

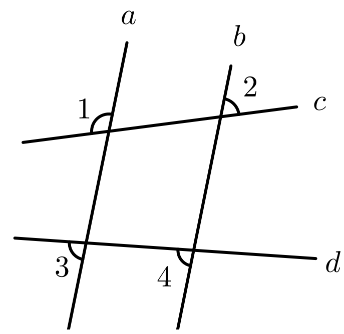
For example, $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 3 = \angle 7$, $\angle 4 = \angle 8$; $\angle 3 = \angle 5$, $\angle 4 = \angle 6$; $\angle 4 + \angle 5 = 180^\circ$, $\angle 3 + \angle 6 = 180^\circ$.

Math Exploration 2

1 As shown in the figure below, $AB//CD$ and $\angle 1 = (4x - 25)^\circ$, $\angle 2 = (85 - x)^\circ$. Find $\angle 1$.

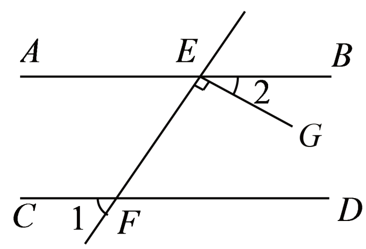


2 As shown in the figure below, $a//b$, which of the following is true? (　　)



A. $\angle 1 = \angle 2$　　B. $\angle 3 + \angle 4 = 180^\circ$　　C. $\angle 2 + \angle 4 = 180^\circ$　　D. $\angle 1 + \angle 2 = 180^\circ$

3 As shown in the figure below, $AB//CD$ and line EF meets AB and CD at E, F, respectively. $EG \perp EF$ at E, if $\angle 1 = 60^\circ$, then the measure of $\angle 2$ is (　　).

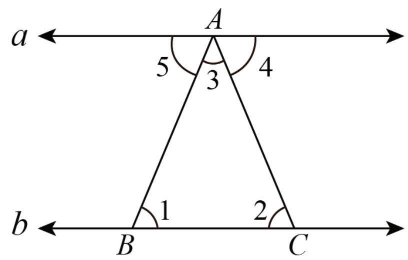


A. 15°　　B. 30°　　C. 45°　　D. 60°

Concept 3: Angles of Triangles

1. Interior angles of the triangle: Angles which are formed by two sides of the triangle. A triangle has three interior angles on its three vertices.

The interior angle sum of a triangle is $180°$.



Proof:

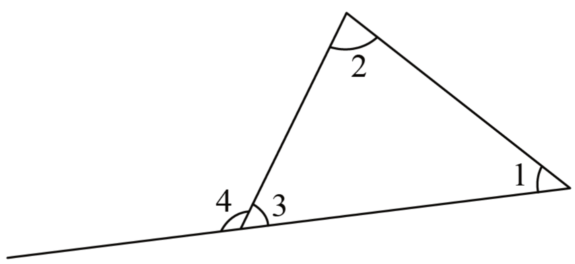
Draw line a such that $a//b$ and line a intersects $\triangle ABC$ at point A. Side BC is on line b.

$\angle 3 + \angle 4 + \angle 5 = 180°$

$\angle 1 = \angle 5, \angle 2 = \angle 4$ as they are alternate interior angles.

Thus, $\angle 1 + \angle 2 + \angle 3 = 180°$, meaning that the interior angle sum of $\triangle ABC$ is $180°$.

2.



Exterior angles of the triangle: Angles which are formed by one side of the triangle and the extension of another side.

Example: $\angle 4$

Remote interior angles: Interior angles which are not adjacent to the exterior angle.

Example: $\angle 1$ and $\angle 2$ are remote angles to $\angle 4$.

Angle relationships between these angles:

$\angle 1 + \angle 2 + \angle 3 = 180°$

$\angle 3 + \angle 4 = 180°$

We can subtract these two equations.

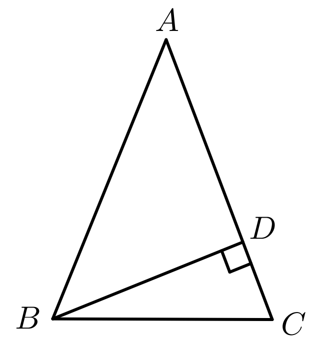
$\angle 4 = \angle 1 + \angle 2$

So we can summarize that the measure of an exterior angle is equal to the sum of its remote angles, and this is called The Exterior Angle Theorem.

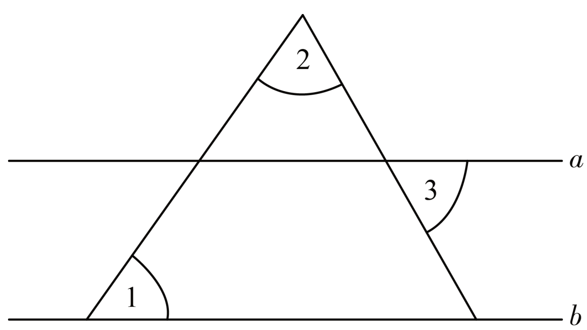
Math Exploration 3

1. In $\triangle ABC$, $\angle A = 35^\circ$, $\angle B = 90^\circ$ then the measure of $\angle C$ is ( ) .
A. 55° B. 65° C. 75° D. 85°

2. As shown in the figure below, in $\triangle ABC$, $\angle C = \angle ABC = 2\angle A$. BD is perpendicular to AC, then $\angle DBC =$ ______ .

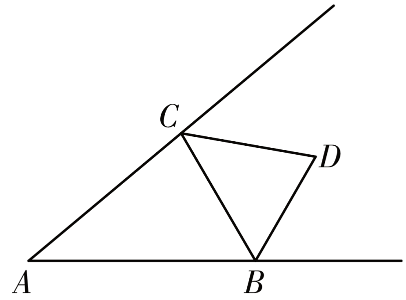


3. As shown in the figure below, $a//b$, $\angle 1 = 55^\circ$, $\angle 2 = 65^\circ$, then the measure of $\angle 3$ is ( ) .

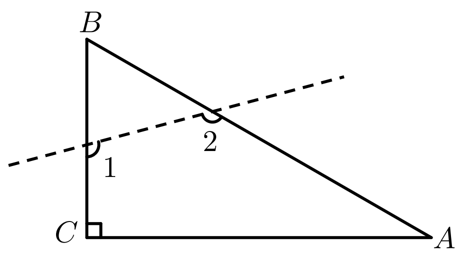


A. 50° B. 55° C. 60° D. 65°

4 As shown in the figure below, given that the exterior angle bisectors of $\angle ABC$ and $\angle ACB$ meet at D, $\angle A = 40^\circ$. Find $\angle D$.

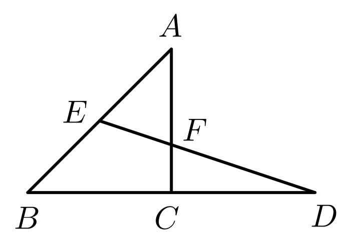


5 In right triangle $\triangle ABC$, $\angle B = 60^\circ$, then $\angle 1 + \angle 2$ is ( ).



A. 150° B. 180° C. 240° D. 300°

6 As shown in the figure below, $AC \perp BD$ at C, given that $\angle A = 40^\circ$ and $\angle AEF = 70^\circ$, then the measure of $\angle D$ is ( ).



A. 20° B. 30° C. 40° D. 50°

Concept 4: Find Angles in Complicated Figures

Steps to find the angle in complicated figures:

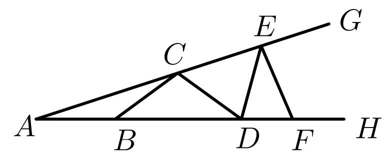
1. Observe the figure and suppose an angle be x

2. Use the properties of parallel lines and triangles to find the relationship between different angles

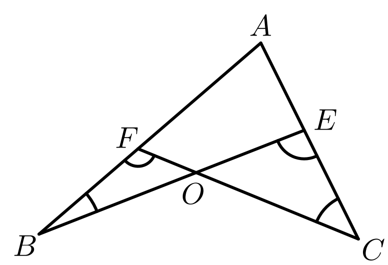
3. Make an equation based on the relationship and find x

Math Exploration 4

1 As shown in the figure below, C, E and B, D, F are on the two sides of $\angle GAH$, respectively. $AB = BC = CD = DE = EF$, if $\angle A = 18^\circ$, find $\angle GEF$.



2 As shown in the figure below, point E is on AC and point F is on AB. BE and CF intersect at O and $\angle C = 2\angle B$, $\angle BFC - \angle BEC = 20^\circ$. Find $\angle C$.



Concept 1: The Pythagorean Theorem

1. The Pythagorean Theorem:

In a right triangle, the two sides that form the right angle are the legs, the other side is called hypotenuse.

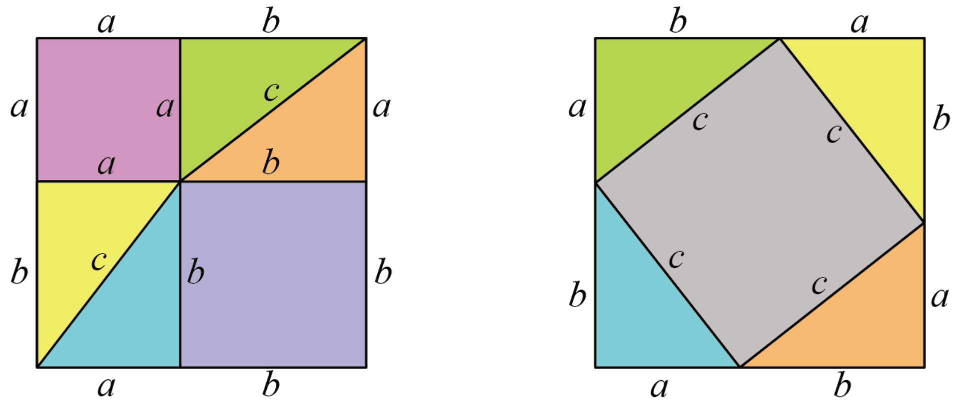
The Pythagorean Theorem is that, in a right triangle, two lags, a, b, and hypotenuse c satisfy that $a^2 + b^2 = c^2$.

We can prove it in this way:

(1) Firstly, draw a right triangle whose longer leg is b and shorter leg is a. c is the hypotenuse.

(2) Use four right triangles to form two squares as shown below.

(3) Since the area of square 1 is the same as that of square 2, we can get $a^2 + b^2 + 2ab = 2ab + c^2$, $a^2 + b^2 = c^2$.



2. The converse of the Pythagorean Theorem

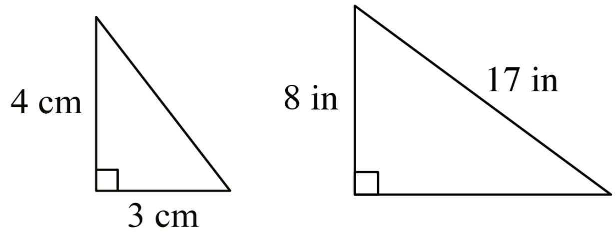
The converse of the Pythagorean Theorem mentions that if the side lengths of a triangle satisty $a^2 + b^2 = c^2$, the triangle is a right triangle.

3. As the picture shown below, for example, if we want to find the distance between $A(x_1, y_1)$ and $B(x_2, y_2)$. we can find the point $C(x_2, y_1)$.

Then we can get $AC = |x_2 - x_1|$, $BC = |y_2 - y_1|$. According to the Pythagorean Theorem, we can get $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Math Exploration 1

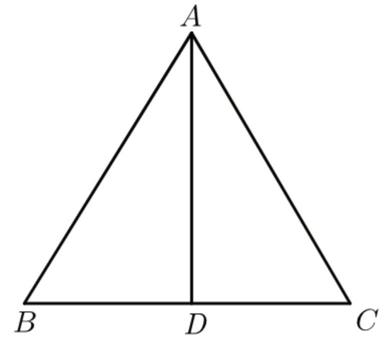
1. Find the length of the missing side:



2. In $\triangle ABC$, $AB = 12\text{cm}$, $BC = 16\text{cm}$ and $AC = 20\text{cm}$, then the area of $\triangle ABC$ is ( ) .
A. 96cm^2 B. 120cm^2 C. 160cm^2 D. 200cm^2

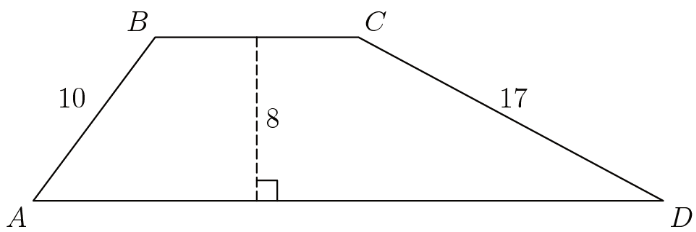
3. Given that the coordinates of point P are $(9, -4)$ and $Q(2, 4)$, then the distance from P to point Q is ______ .

4 As shown in the figure below, in $\triangle ABC$, $AB = 10$, $BC = 12$, D is the midpoint of BC and $AD = 8$. The length of AC is (   ).



A. 10 B. 12 C. $2\sqrt{34}$ D. $4\sqrt{13}$

5 The area of trapezoid $ABCD$ is 164cm^2. The altitude is 8cm, AB is 10cm, and CD is 17cm. What is BC, in centimeters? (2003 AMC 8 Problems, Question #21)



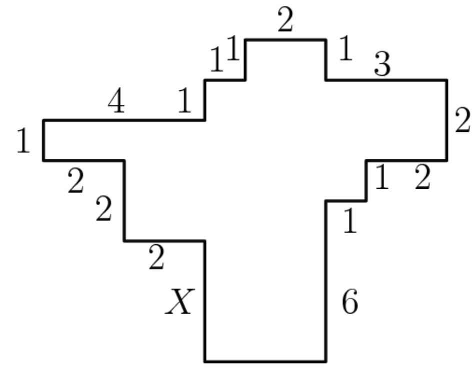
A. 9 B. 10 C. 12 D. 15 E. 20

Concept 2: Perimeter

The perimeter of a two-dimensional shape is the distance around the shape. We can use translation to find the perimeter of a complicated figure.

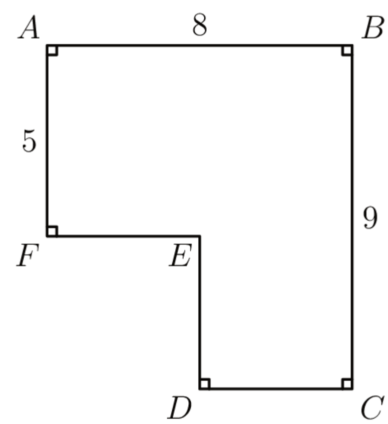
Math Exploration 2

1. In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is, X in centimeters? (2012 AMC 8 Problem, Question #5)



A. 1 B. 2 C. 3 D. 4 E. 5

2. The area of polygon $ABCDEF$ is 52 with $AB = 8, BC = 9$ and $FA = 5$. What is $DE + EF$? (2005 AMC 8 Problem, Question #13)



A. 7 B. 8 C. 9 D. 10 E. 11

Concept 3: Find Length in Triangles (Similar Figure)

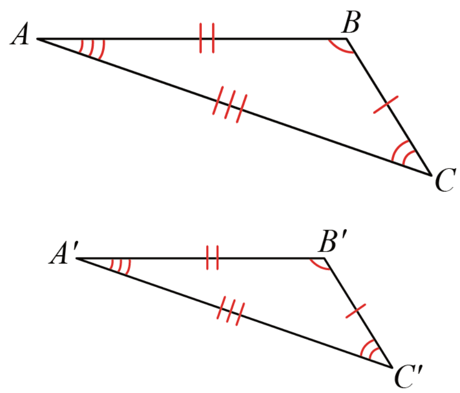
1. **Similar figures** have the same shape but may have different sizes.

If two figures are **similar** ($\triangle ABC \sim \triangle A'B'C'$), we have:

(1) their corresponding angles have the same measure ($\angle A = \angle A'$, $\angle B = \angle B'$, $\angle C = \angle C'$)

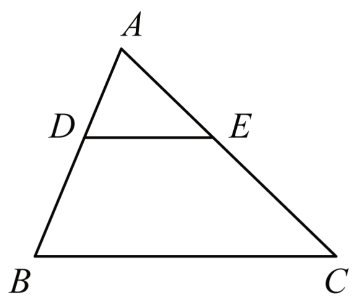
(2) the lengths of their corresponding sides are proportional
$(\frac{AB}{A'B'} = \frac{AC}{A'C'} = \frac{BC}{B'C'} = \frac{\text{Perimeter}ABC}{\text{Perimeter}A'B'C'})$.

(3) the ratio of the area of two similar triangles is equal to the square of the ratio of any pair of the corresponding sides of the similar triangles.
$(\frac{\text{Area } \triangle ABC}{\text{Area } \triangle A'B'C'} = \frac{AB^2}{A'B'^2} = \frac{AC^2}{A'C'^2} = \frac{BC^2}{B'C'^2})$

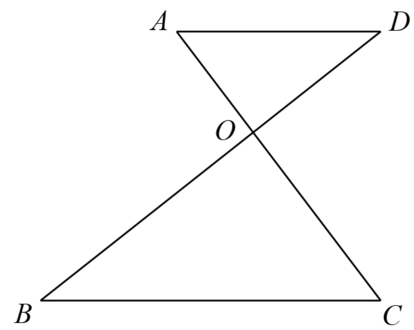


2. **Angle-Angle (AA) Similarity Postulate**: If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

3. "A" model: If $DE//BC$, we have $\triangle ADE \sim \triangle ABC$, and then $\frac{AD}{AB} = \frac{AE}{AC} = \frac{DE}{BC}$.

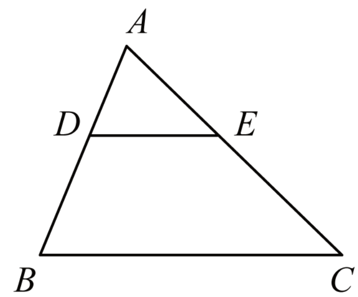


"8" model: If $AD//BC$, we have $\triangle OAD \sim \triangle OCB$, and then $\dfrac{AD}{CB} = \dfrac{AO}{CO} = \dfrac{BO}{DO}$.

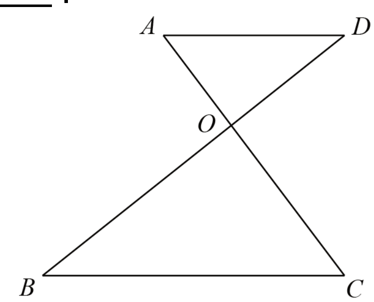


Math Exploration 3

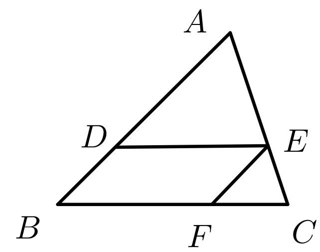
1. Given that $\overline{DE}//\overline{BC}$, $AE : AC = 3 : 7$, $AD = 15, AC = 60$, $AE =$ ______ , $AB =$ ______ .



2. Given that $\overline{AD}//\overline{BC}$, $OA = 12$, $OC = 18$, and $AD = 24$, $BC =$ ______ .

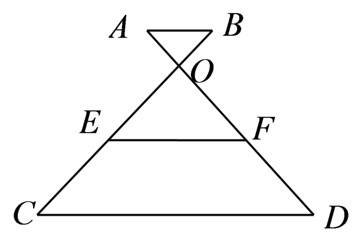


3 As shown in the figure below, in $\triangle ABC$, point D, E and F are on side AB, AC and BC, respectively and $DE//BC$, $EF//AB$. If $AD = 2BD$, then $\dfrac{CF}{CB}$ is ( ) .



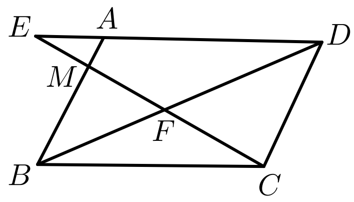
A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{2}{3}$

4 As shown in the figure below, $AB//CD//EF$, $BO : OC = 1 : 4$, points E and F are midpoints of OC and OD, respectively. $EF : AB$ is ( ) .



A. 1 B. 2 C. 3 D. 4

5 As shown in the figure below, quadrilateral $ABCD$ is a parallelogram and point E is on line DA. Given that $AE = 2, AD = 8$ and $AM = 1$, find AB.



Concept 1: Areas of Basic Figures

Area of square: $A = l^2$

Area of rectangle: $A = lw$

Area of parallelogram: $A = bh$

Area of trapezoid: $A = \frac{1}{2}(b_1 + b_2)h$

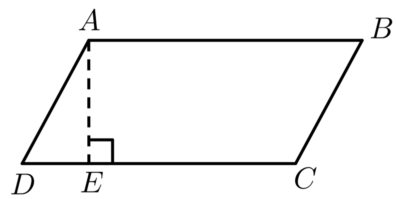
Area of triangle: $A = \frac{1}{2}bh$

Area of rhombus: $A = \frac{1}{2}d_1 d_2$

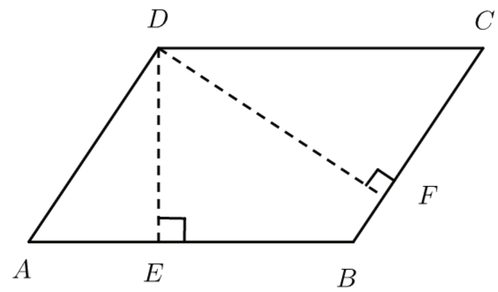
Area of circle: $A = \pi r^2$

Math Exploration 1

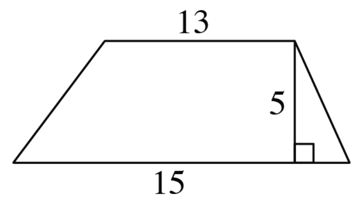
1 (1) In parallelogram $ABCD$, $\overline{AE}$ is perpendicular to $\overline{DC}$ at point E. $CD = 18, AE = 7$. The area of parallelogram $ABCD$ is ______ .



(2) In parallelogram $ABCD$, $\overline{DE}$ is perpendicular to $\overline{AB}$ at point E and $\overline{DF}$ is perpendicular to $\overline{BC}$ at point F. $AB = 21, DE = 16, DF = 24$, then $BC =$ ______ .



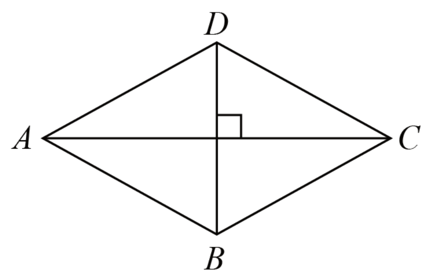
2 The area of the following trapezoid is ______ .



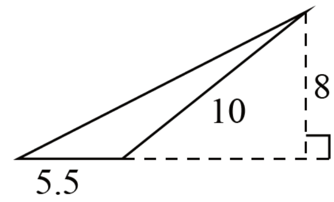
3 In rhombus $ABCD$, diagonal $AC = 48$ and diagonal $BD = 14$.

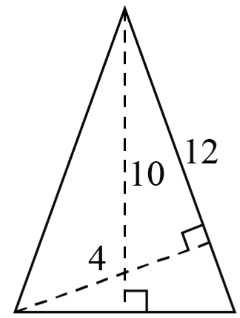
The area of rhombus $ABCD$ is ______ .

The side length of rhombus $ABCD$ is ______ .



4 Find the area of the triangles below.





5 1. If the height of a triangle is 12, and the area of the triangle is 36, the length of the corresponding base is ______ .

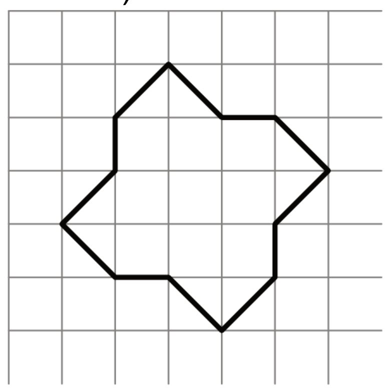
2. If the base of a triangle is 5, and the area of the triangle is 35, the length of the corresponding height is ______ .

Concept 2: Areas of Shapes on Grids

To find the area on grid, cut the figure into several parts or patch them into a whole figure.

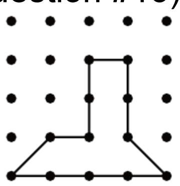
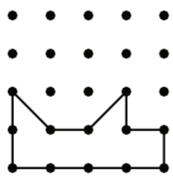
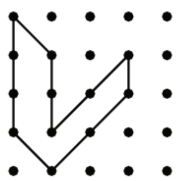
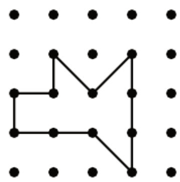
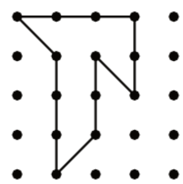
Math Exploration 2

1. The twelve-sided figure shown has been drawn on $1\text{cm} \times 1\text{cm}$ graph paper. What is the area of the figure in cm^2? (   ) . (2018 AMC 8 Problem, Question #4)

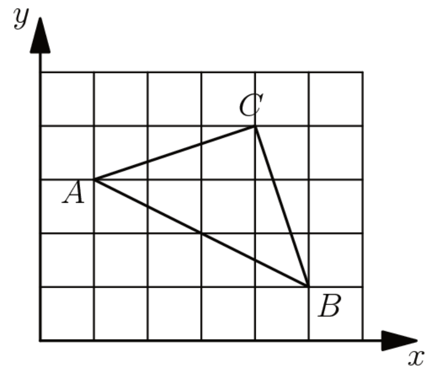


A. 12　　B. 12.5　　C. 13　　D. 13.5　　E. 14

2. Which of the following polygons has the largest area? (   ) . (2002 AMC 8 Problem, Question #15)

A. 　B. 　C. 　D. 　E. 

3 A triangle with vertices as $A=(1,3)$, $B=(5,1)$, and $C=(4,4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle? (　　) . (2015 AMC 8 Problem, Question #19)

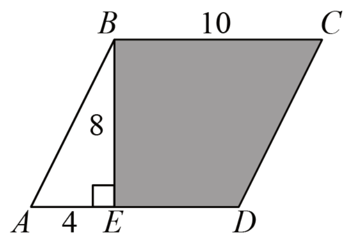


A. $\frac{1}{6}$　　B. $\frac{1}{5}$　　C. $\frac{1}{4}$　　D. $\frac{1}{3}$　　E. $\frac{1}{2}$

Concept 3: Areas of Complicated Figures

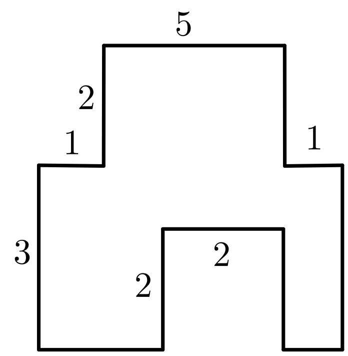
Math Exploration 3

1. The area of the shaded region $BEDC$ in parallelogram $ABCD$ is ( ) . (Adapted from 1989 AJHSME Problem, Question #15)



A. 24 B. 48 C. 60 D. 64 E. 80

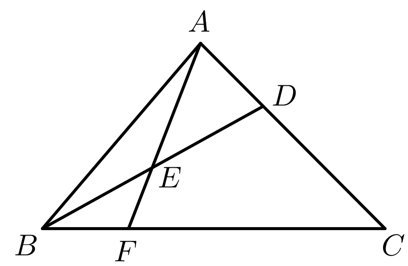
2. Find the area of the figure below.



3 A 1×2 rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle? (   ) . (2013 AMC 8 Problem, Question #20)

A. $\frac{\pi}{2}$ B. $\frac{2\pi}{2}$ C. π D. $\frac{4\pi}{3}$ E. $\frac{5\pi}{3}$

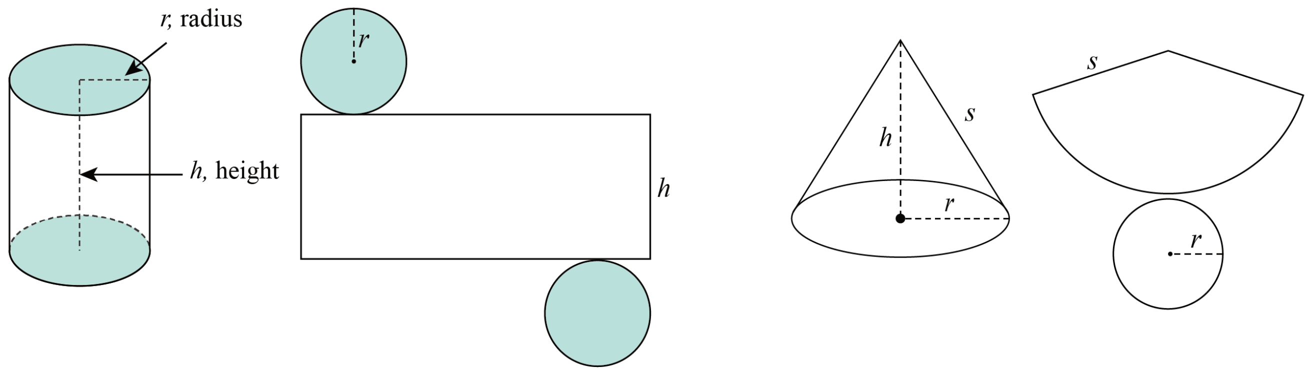
4 In triangle ABC, point D divides side $\overline{AC}$ so that $AD : DC = 1 : 2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$? (2019 AMC 8 Problem, Question #24)



A. 24 B. 30 C. 32 D. 36 E. 40

Lesson 4 Volume

Concept 1: Volume of Cylinders and Cones



A cylinder is a solid with two congruent circular bases that lie in parallel lines.

Volume $V = Bh = \pi r^2 h$

A cone is a solid with a circular base.

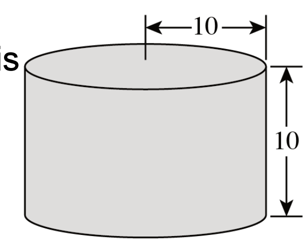
The slant height s of a cone is the distance between any point on the edge of the base and the vertex.

Slant height $s = \sqrt{r^2 + h^2}$

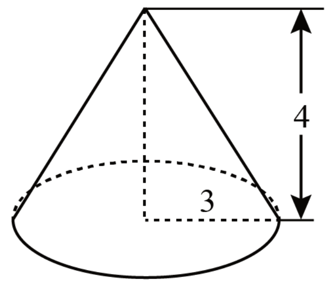
Volume $V = \dfrac{1}{3}\pi r^2 h$

Math Exploration 1

1 Answer the questions (leave π in the answer):
The height of a cylinder is 10, and the radius of the circular base is also 10.
The volume of the cylinder is ______ .



2 Find the volume of the following cone. (keep π in the answer)



3 (1) If the volume of a cylinder is 375π and the height is 15π, the radius is ______ .

(2) If the surface area of a cylinder is 128π, and the radius is 4, the height is ______ .

4 (1) The cylinder and the cone have the same base and the same volume. If the height of the cylinder is 3 cm, then the height of the cone is ______ cm.

(2) The cylinder and the cone have the same volume and the same height. If the radius of the base of cylinder is 3 cm, then the radius of the base of cone is ______ cm.

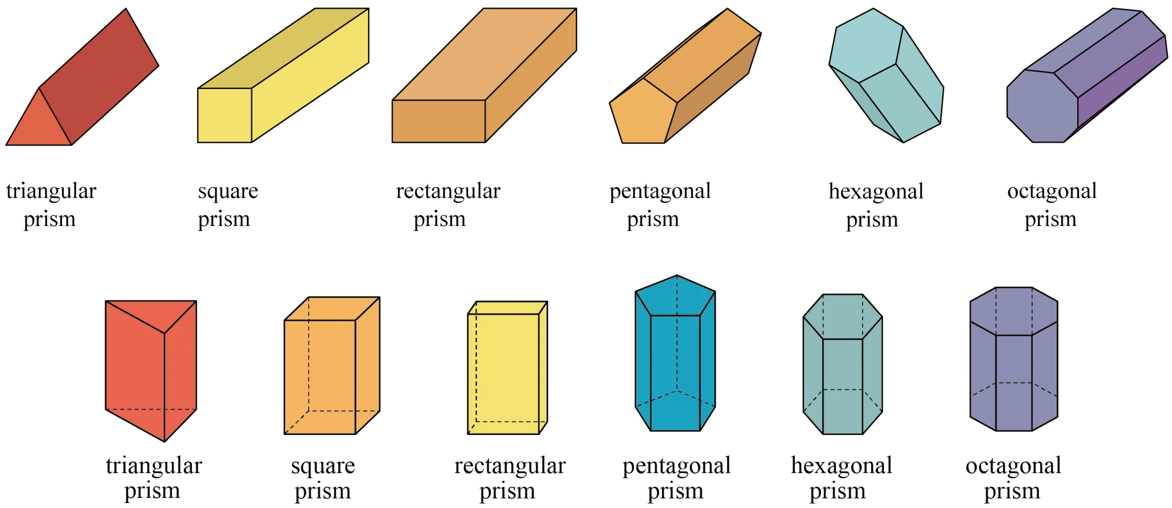
5. Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 8 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 8 cm high. What is the ratio of the volume one of Alex's cans to the volume one of Felicia's cans?

6. A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased? (2004 AMC 12A Problem, Question #9)

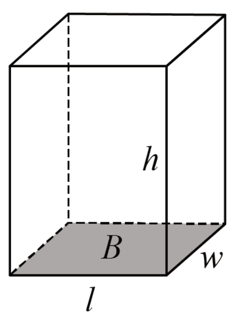
A. 10 B. 25 C. 36 D. 50 E. 60

Concept 2: Volume of Prisms and Pyramids

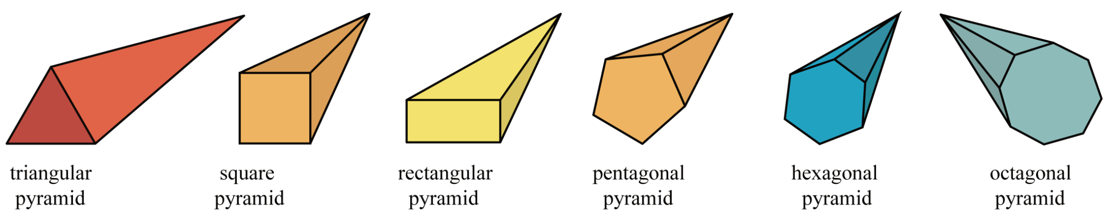
1. Definition of Prism: A prism is a three-dimensional figure with two identical and parallel bases that are polygons and the other faces are rectangles. A prism is identified by the shape and its base.

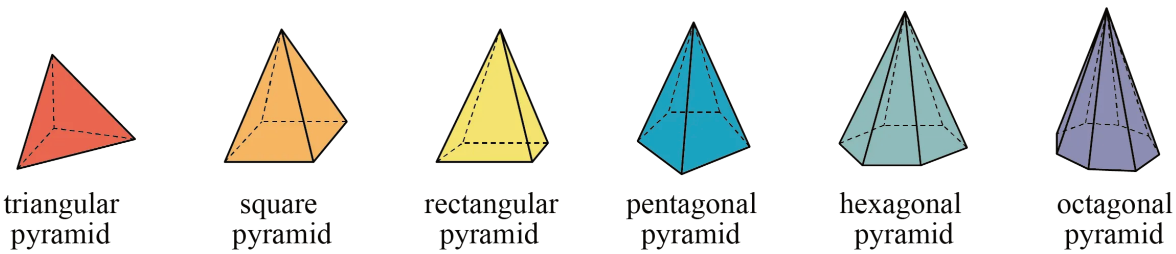


2. Volume of prisms:
$V = Bh$

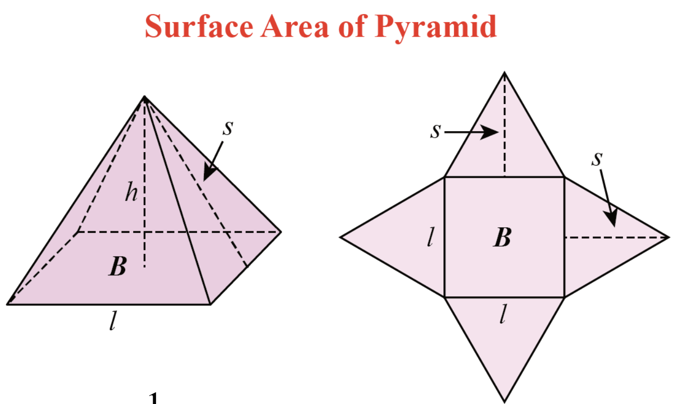


3. A pyramid is a three-dimensional figure whose base is a polygon and whose other faces are triangles that meet at a point. A pyramid is identified by the shape of its base.





4. Volume of pyramids:

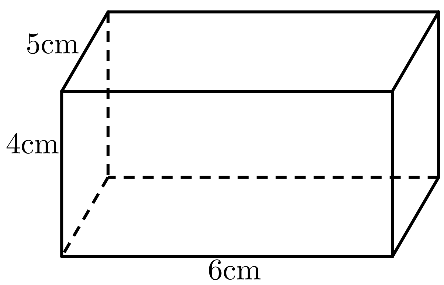


The volume is calculated by $\frac{1}{3} \times B \times h$.

Math Exploration 2

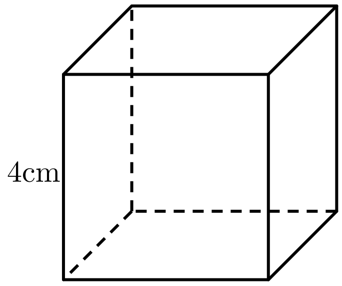
1 Find the volumes of the following solid figures:

(1)



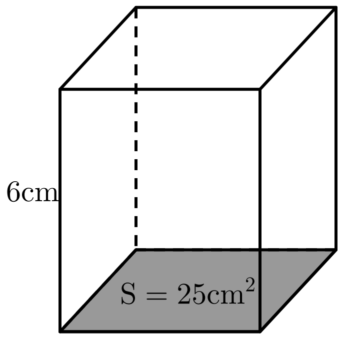
$V =$ ______________________ cm^3.

(2)



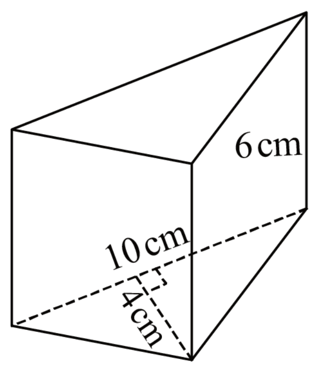
V = ________________________ cm^3.

(3)



V = ________________________ cm^3.

2 Find the volume of the following triangular prism:



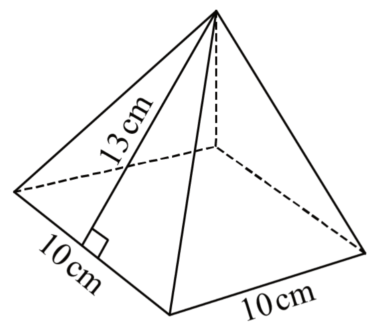
3 Find the surface area of the square pyramid.

The area of the base is ______ cm^2.

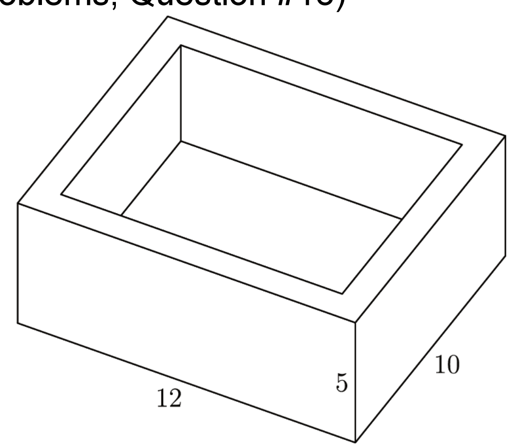
The area of the four faces is ______ cm^2.

The surface area is ______ cm^2.

The volume is ______ cm^3.

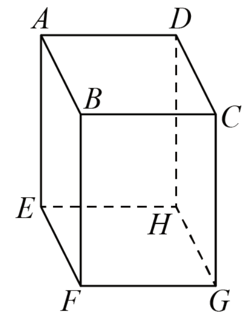


4 Isabella uses one-foot cubical blocks to build a rectangular fort that is 12 feet long, 10 feet wide, and 5 feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain? (　　) . (2018 AMC 8 Problems, Question #18)



A. 204　　B. 280　　C. 320　　D. 340　　E. 600

5 In following rectangular prism, $AB = 4, BC = 3, BF = 6$. Find the volume of triangular prism $ABC - EFG$ and triangular pyramid $A - EFG$ and the length of AG.



Learning Modules

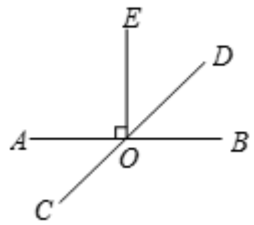
Lesson 1

1 As shown in the figure below, line AB meets line CD at O. $OE \perp AB$ and $\angle BOD = 45°$.

(1) $\angle COE =$ ______ °, it is an ______ (obtuse/acute) angle.

(2) $\angle COA =$ ______ °, it is an ______ (obtuse/acute) angle.

(3) $\angle COA$ and $\angle COE$ are ______ (complementary/supplementary) angles.



Answer 1:135

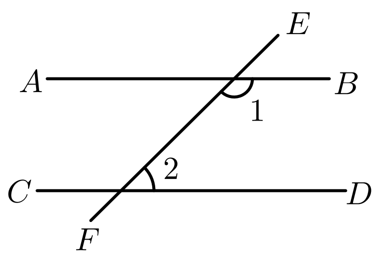
2:obtuse

3:45

4:acute

5:supplementary

2 As shown in the figure below, $AB // CD$ and $\angle 1 = (4x - 25)°$, $\angle 2 = (85 - x)°$. Find $\angle 1$.



Answer $135°$

Solution

$\because AB//CD$,

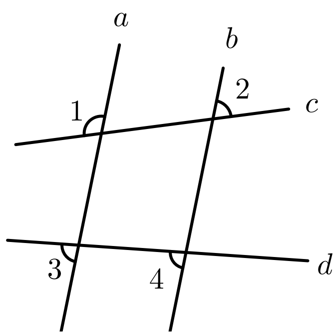
$\therefore \angle 1+\angle 2=180^{\circ}$,

we can get $(4x-25)+(85-x)=180$,

then $x=40$.

$\therefore \angle 1=(4x-25)^{\circ}=135^{\circ}$.

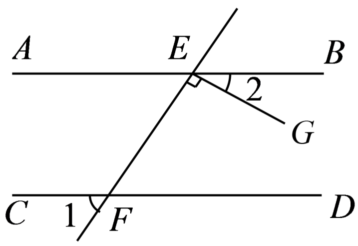
3 As shown in the figure below, $a//b$, which of the following is true? (   )



A. $\angle 1=\angle 2$ B. $\angle 3+\angle 4=180^{\circ}$ C. $\angle 2+\angle 4=180^{\circ}$ D. $\angle 1+\angle 2=180^{\circ}$

Answer D

4 As shown in the figure below, $AB//CD$ and line EF meets AB and CD at E, F, respectively. $EG\perp EF$ at E, if $\angle 1=60^{\circ}$, then the measure of $\angle 2$ is (   ).



A. 15° B. 30° C. 45° D. 60°

Answer B

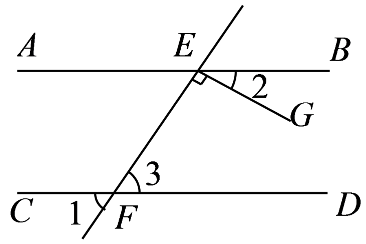
Solution $\angle 3=\angle 1=60^{\circ}$,

$\because AB//CD$, $EG\perp EF$,

$\therefore \angle 3+90^{\circ}+\angle 2=180^{\circ}$,

$60^{\circ}+90^{\circ}+\angle 2=180^{\circ}$,

$\angle 2=30^{\circ}$.

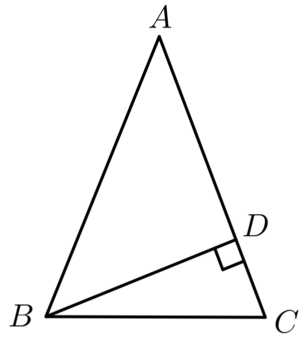


5 In $\triangle ABC$, $\angle A=35^{\circ}$, $\angle B=90^{\circ}$ then the measure of $\angle C$ is ( ) .

A. 55° B. 65° C. 75° D. 85°

Answer A

6 As shown in the figure below, in $\triangle ABC$, $\angle C=\angle ABC=2\angle A$. BD is perpendicular to AC, then $\angle DBC=$ ______ .



Answer 18°

Solution Suppose $\angle A=x$, then $\angle C=\angle ABC=2x$.

$\angle C+\angle ABC+\angle A=180^{\circ}$,

so $2x+2x+x=180^{\circ}$,

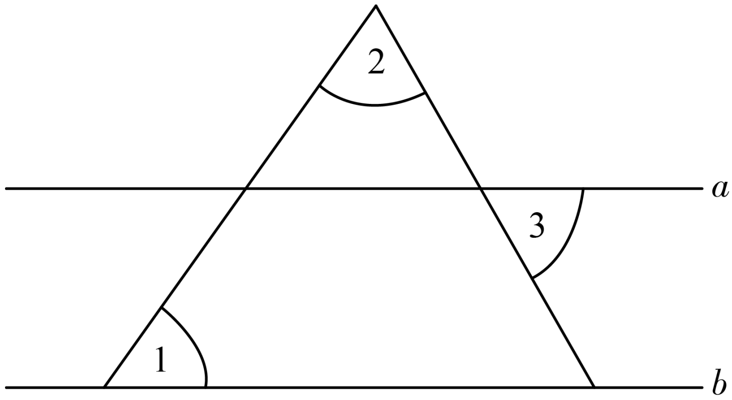
therefore $x=36^{\circ}$, $\angle C=2x=72^{\circ}$.

In right triangle BDC, $\angle DBC=90^{\circ}-\angle C=90^{\circ}-72^{\circ}=18^{\circ}$.

7

As shown in the figure below, $a//b$, $\angle 1=55^{\circ}$, $\angle 2=65^{\circ}$, then the measure of $\angle 3$ is（

）.



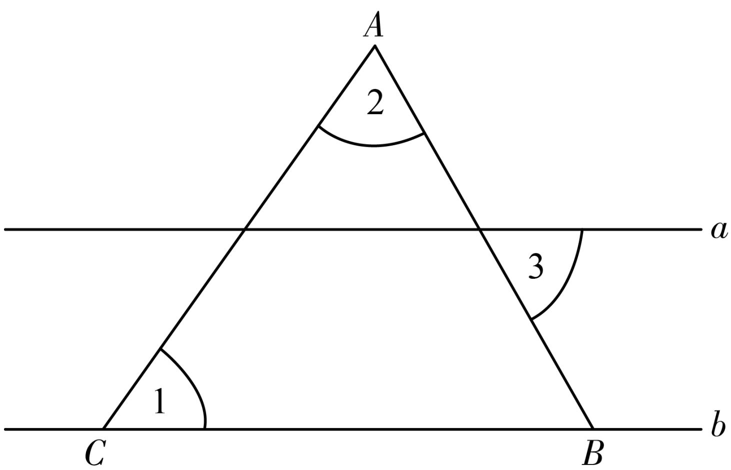
A. 50° B. 55° C. 60° D. 65°

Answer C

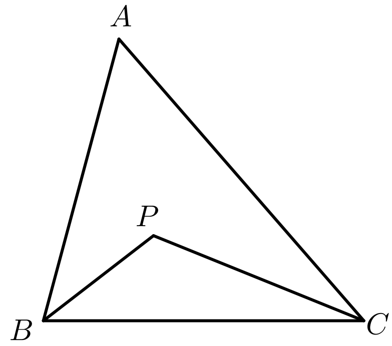
Solution As shown in the figure below,

$\because \angle 1=55^{\circ}$, $\angle 2=65^{\circ}$, $\therefore \angle ABC=60^{\circ}$.

$\because a//b$, $\therefore \angle 3=\angle ABC=60^{\circ}$.



8 As shown in the figure below, in $\triangle ABC$, $\angle BAC=50^{\circ}$, BP bisects $\angle ABC$ and CP bisects $\angle ACB$. Then $\angle BPC$ is ______ °.



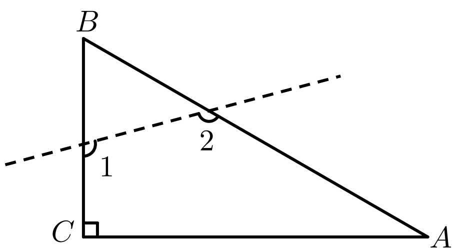
Answer 115

Solution $\angle BPC = \angle A + \angle ABP + \angle ACP$

$= (180^\circ - \angle ABC - \angle ACB) + \frac{1}{2}(\angle ABC + \angle ACB)$

$= 115^\circ$

9 In right triangle $\triangle ABC$, $\angle B = 60^\circ$, then $\angle 1 + \angle 2$ is (   ).

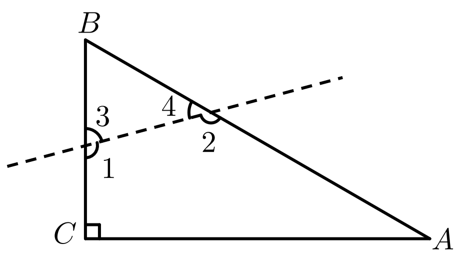


A. 150° B. 180° C. 240° D. 300°

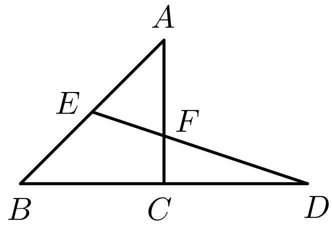
Answer C

Solution $\angle 1 = \angle B + \angle 4$, $\angle 2 = \angle 3 + \angle B$,

$\therefore \angle 1 + \angle 2 = \angle 3 + \angle 4 + 2\angle B = 180^\circ + \angle B = 240^\circ$.



10 As shown in the figure below, $AC \perp BD$ at C, given that $\angle A = 40^\circ$ and $\angle AEF = 70^\circ$, then the measure of $\angle D$ is (   ).



A. 20° B. 30° C. 40° D. 50°

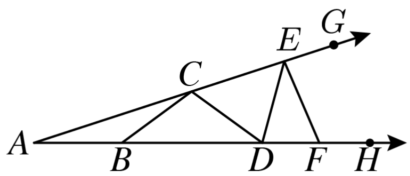
Answer A

Solution $\angle AFE = 180^\circ - \angle A - \angle AEF = 180^\circ - 40^\circ - 70^\circ = 70^\circ$,

$\angle CFD = \angle AFE = 70^\circ$,

$\angle D = 180^\circ - \angle CFD - \angle ACD = 180^\circ - 70^\circ - 90^\circ = 20^\circ$.

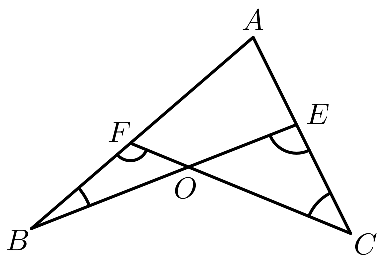
11 As shown in the figure below, C, E and B, D, F are on the two sides of $\angle GAH$, respectively. $AB = BC = CD = DE = EF$, if $m\angle A = 18^\circ$, find $m\angle GEF$.



Answer $m\angle GEF = 90^\circ$

Solution $m\angle A = 18^\circ \to m\angle C = 18^\circ \to m\angle ABC = 180^\circ - 18^\circ - 18^\circ = 144^\circ \to m\angle CBD = 36^\circ \to m\angle BCD = 180^\circ - 36^\circ - 36^\circ = 108^\circ \to m\angle ECD = 180^\circ - 18^\circ - 108^\circ = 54^\circ \to m\angle EDC = 72^\circ \to m\angle EDF = m\angle EFD = 180^\circ - 72^\circ - 36^\circ = 72^\circ \to m\angle AEF = 180^\circ - 18^\circ - 72^\circ = 90^\circ \to m\angle GEF = 90^\circ$

12 As shown in the figure below, point E is on AC and point F is on AB. BE and CF intersect at O and $\angle C = 2\angle B$, $\angle BFC - \angle BEC = 20^\circ$. Find $\angle C$.



Answer 40°

Solution $\because \angle BFC = \angle A + \angle C$,

$\angle BEC = \angle A + \angle B$,

$\therefore \angle BFC - \angle BEC = \angle C - \angle B = 20^{\circ}$,

$\because \angle C = 2\angle B$,

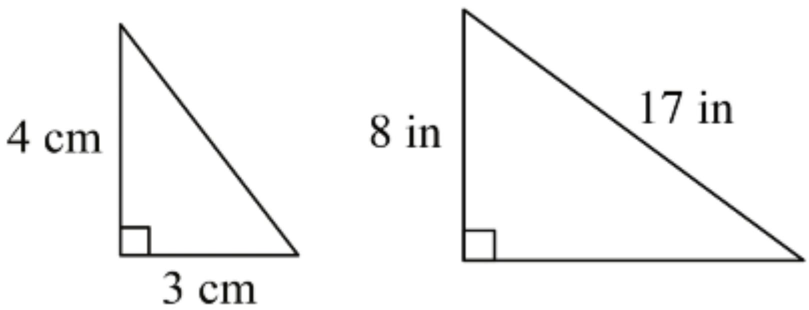
$\therefore 2\angle B - \angle B = 20^{\circ}$,

$\therefore \angle B = 20^{\circ}$,

$\therefore \angle C = 2\angle B = 40^{\circ}$.

Lesson 2

13 Find the length of the missing side:



Answer 5 cm; 8 inches.

14 In $\triangle ABC$, $AB = 12\text{cm}$, $BC = 16\text{cm}$ and $AC = 20\text{cm}$, then the area of $\triangle ABC$ is ( ) .

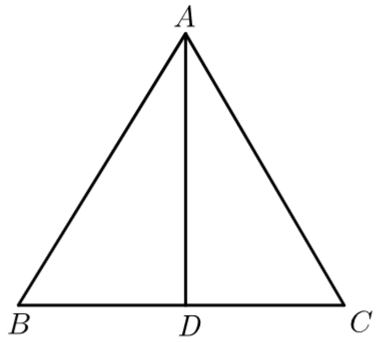
A. $96\mathrm{cm}^2$ B. $120\mathrm{cm}^2$ C. $160\mathrm{cm}^2$ D. $200\mathrm{cm}^2$

Answer A

15 Given that the coordinates of point P are $(9,-4)$ and $Q(2,4)$, then the distance from P to point Q is ______ .

Answer $\sqrt{113}$

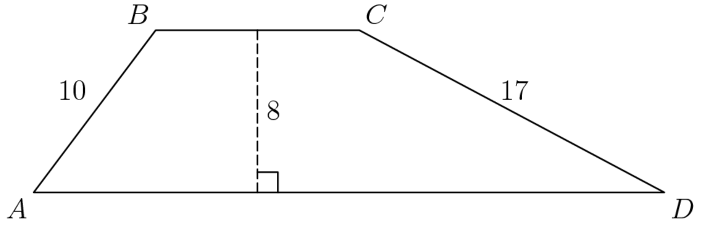
16 As shown in the figure below, in $\triangle ABC$, $AB=10$, $BC=12$, D is the midpoint of BC and $AD=8$. The length of AC is ( ).



A. 10 B. 12 C. $2\sqrt{34}$ D. $4\sqrt{13}$

Answer A

17 The area of trapezoid $ABCD$ is $164\mathrm{cm}^2$. The altitude is $8\mathrm{cm}$, AB is $10\mathrm{cm}$, and CD is $17\mathrm{cm}$. What is BC, in centimeters? (2003 AMC 8 Problems, Question #21)

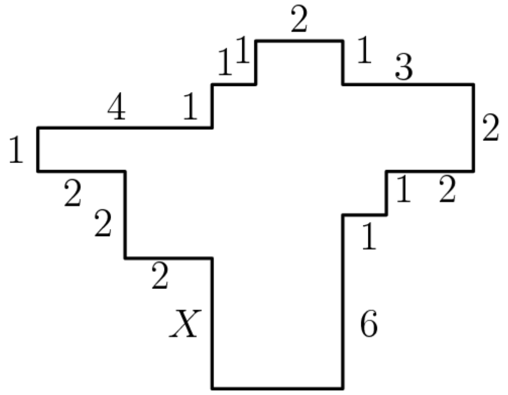


A. 9 B. 10 C. 12 D. 15 E. 20

Answer B

Solution Using the formula for the area of a trapezoid, we have $164 = 8\left(\frac{BC + AD}{2}\right)$. Thus $BC + AD = 41$. Drop perpendiculars from B to AD and from C to AD and let them hit AD at E and F respectively. Note that each of these perpendiculars has length 8 . From the Pythagorean Theorem, $AE = 6$ and $DF = 15$ thus $AD = BC + 21$. Substituting back into our original equation we have $BC + BC + 21 = 41$ thus $BC =$(B) 10.

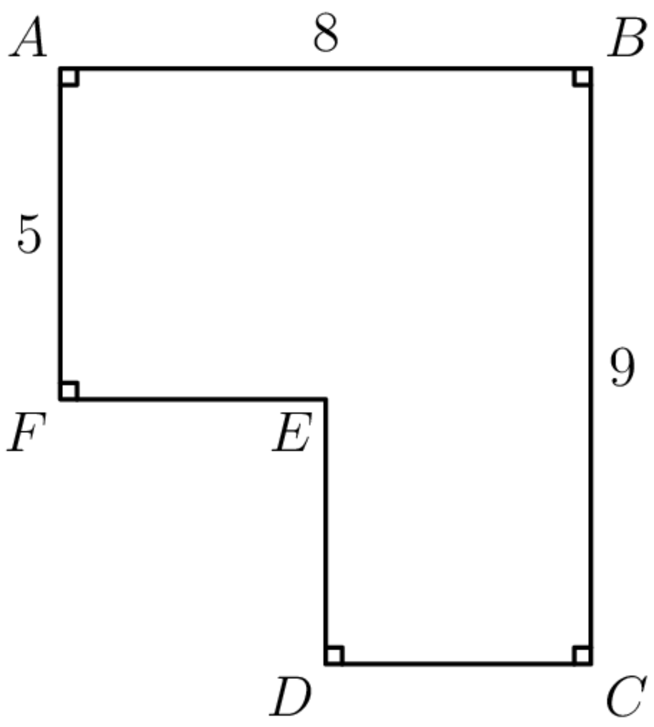
18 In the diagram, all angles are right angles and the lengths of the sides are given in centimeters. Note the diagram is not drawn to scale. What is, X in centimeters? (2012 AMC 8 Problem, Question #5)



A. 1 B. 2 C. 3 D. 4 E. 5

Answer E

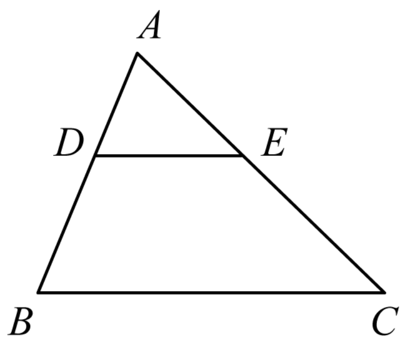
19 The area of polygon $ABCDEF$ is 52 with $AB = 8, BC = 9$ and $FA = 5$. What is $DE + EF$? (2005 AMC 8 Problem, Question #13)



A. 7 B. 8 C. 9 D. 10 E. 11

Answer C

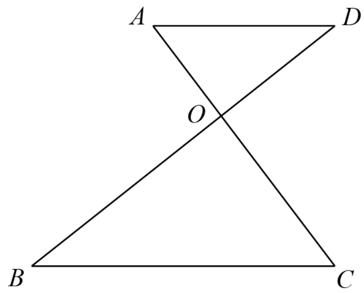
20 Given that $\overline{DE}//\overline{BC}$, $AE:AC=3:7$, $AD=15$, $AC=60$, $AE=$ ______ , $AB=$ ______ .



Answer 1:18

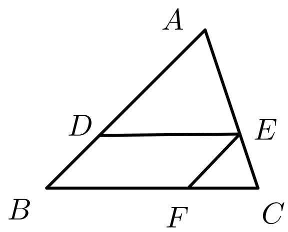
2:50

21 Given that $\overline{AD}//\overline{BC}$, $OA=12$, $OC=18$, and $AD=24$, $BC=$ ______ .



Answer 36

22 As shown in the figure below, in $\triangle ABC$, point D, E and F are on side AB, AC and BC, respectively and $DE//BC$, $EF//AB$. If $AD = 2BD$, then $\dfrac{CF}{CB}$ is ( ) .



A. $\frac{1}{2}$ B. $\frac{1}{3}$ C. $\frac{1}{4}$ D. $\frac{2}{3}$

Answer B

Solution $\because AD = 2BD$,

$\therefore BD : AB = 1 : 3$,

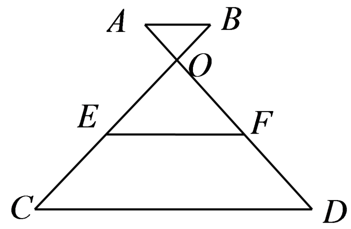
$\because DE//BC$,

$\therefore CE : AC = BD : AB = 1 : 3$,

$\because EF//AB$,

$\therefore CF : CB = CE : AC = 1 : 3$.

23 As shown in the figure below, $AB//CD//EF$, $BO : OC = 1 : 4$, points E and F are midpoints of OC and OD, respectively. $EF : AB$ is ( ) .



A. $1:1$ B. $2:1$ C. $3:1$ D. $4:1$

Answer B

Solution $EF//CD$

$\because AB//CD$,

$\therefore \triangle ABO \backsim \triangle FEO$,

$\therefore EF:AB = EO:BO$

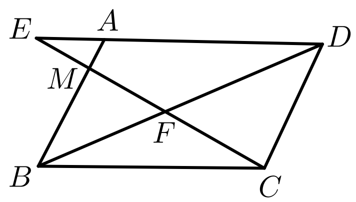
$BO:OC = 1:4$

$\therefore OE = \frac{1}{2}OC$

$\therefore OE = 2OB$

$\therefore EF:AB = 2:1$.

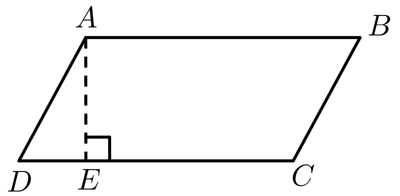
24 As shown in the figure below, quadrilateral $ABCD$ is a parallelogram and point E is on line DA. Given that $AE = 2$, $AD = 8$ and $AM = 1$, find AB .



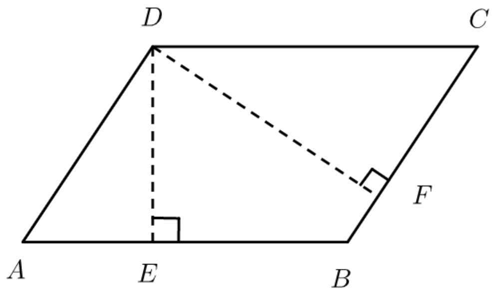
Answer 5

Lesson 3

25 (1) In parallelogram $ABCD$, $\overline{AE}$ is perpendicular to $\overline{DC}$ at point E. $CD = 18, AE = 7$. The area of parallelogram $ABCD$ is ______ .



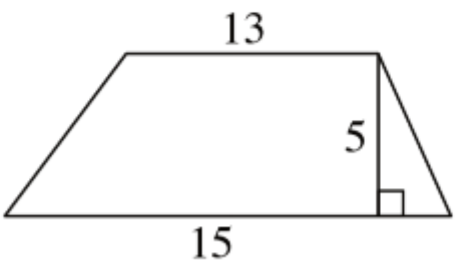
(2) In parallelogram $ABCD$, $\overline{DE}$ is perpendicular to $\overline{AB}$ at point E and $\overline{DF}$ is perpendicular to $\overline{BC}$ at point F. $AB = 21, DE = 16, DF = 24$, then $BC =$ ______ .



Answer 1:126

2:14

26 The area of the following trapezoid is ______ .

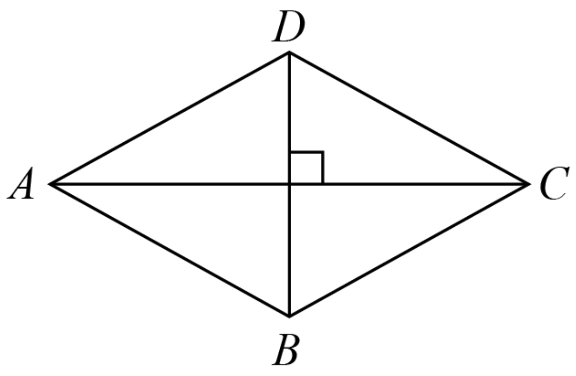


Answer 70

27 In rhombus $ABCD$, diagonal $AC = 48$ and diagonal $BD = 14$.

The area of rhombus $ABCD$ is ______ .

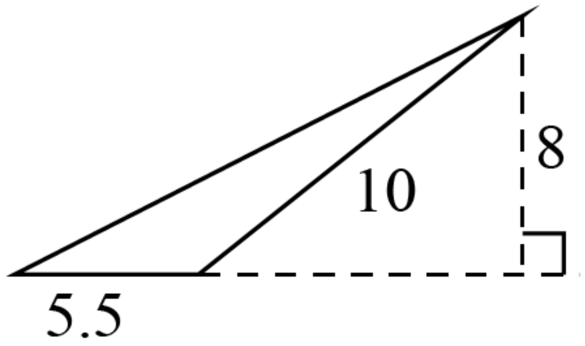
The side length of rhombus $ABCD$ is ______ .

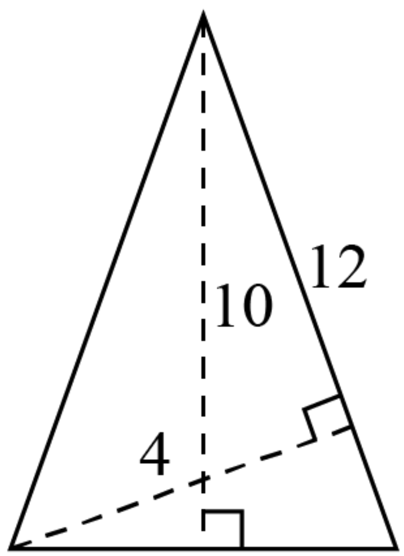


Answer 1:336

2:25

28 Find the area of the triangles below.





Answer 22

24

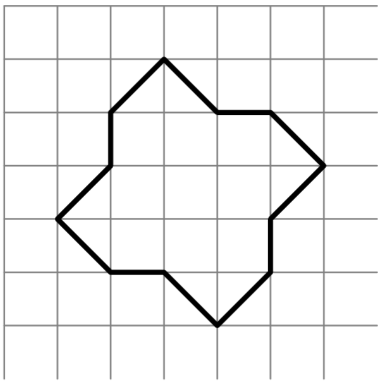
29 1. If the height of a triangle is 12, and the area of the triangle is 36, the length of the corresponding base is ______ .

2. If the base of a triangle is 5, and the area of the triangle is 35, the length of the corresponding height is ______ .

Answer 1:6

2:14

30 The twelve-sided figure shown has been drawn on $1\text{cm} \times 1\text{cm}$ graph paper. What is the area of the figure in cm^2? ( ) . (2018 AMC 8 Problem, Question #4)



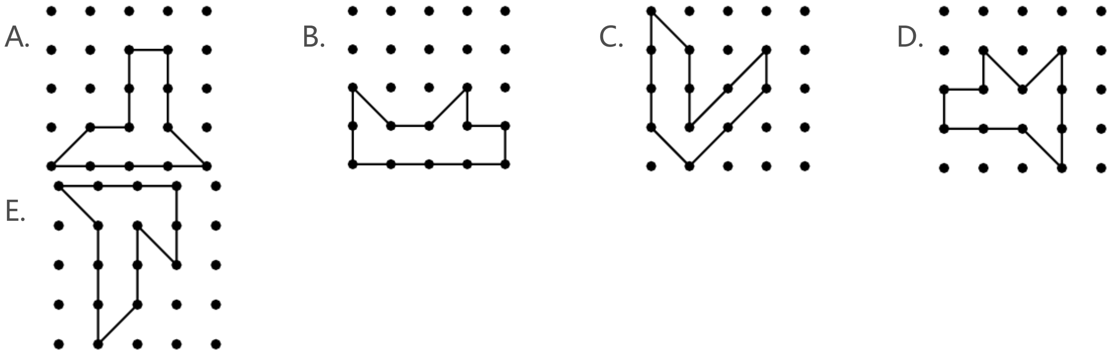
A. 12 B. 12.5 C. 13 D. 13.5 E. 14

Answer C

Solution We count $3 \cdot 3 = 9$ unit squares in the middle, and 4 small triangles each with an area of 1. Thus, the answer is $9 + 4 = \boxed{(C)13}$.

31

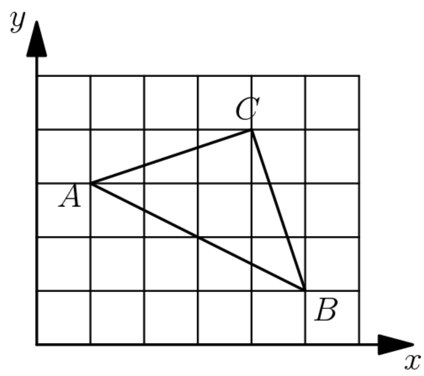
Which of the following polygons has the largest area? (　) . (2002 AMC 8 Problem, Question #15)



Answer E

Solution Each polygon can be divided into unit squares and right triangles with side length 1. Count the number of boxes enclosed by each polygon, with the unit square being 1, and the triangle being 5. A has 5, B has 5, C has 5, D has 4.5, and E has 5.5. Therefore, the polygon with the largest area is E.

32 A triangle with vertices as $A=(1,3)$, $B=(5,1)$, and $C=(4,4)$ is plotted on a 6×5 grid. What fraction of the grid is covered by the triangle? (　) . (2015 AMC 8 Problem, Question #19)



A. $\frac{1}{6}$　　B. $\frac{1}{5}$　　C. $\frac{1}{4}$　　D. $\frac{1}{3}$　　E. $\frac{1}{2}$

Answer A

Solution

The area of $\triangle ABC$ is equal to half the product of its base and height. By the Pythagorean Theorem, we find its height is $\sqrt{1^2+2^2}=\sqrt{5}$, and its base is $\sqrt{2^2+4^2}=\sqrt{20}$. We muliply these and divide by 2 to find the of the triangle is $\frac{\sqrt{5\cdot 20}}{2}=\frac{\sqrt{100}}{2}=\frac{10}{2}=5$. Since the grid has an area of 30, the fraction of the grid covered by the triangle $\frac{5}{30}=\boxed{(\text{A})\frac{1}{6}}$.

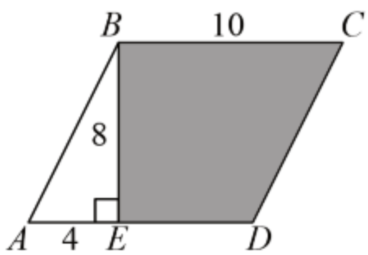
Note ange $\angle ACB$ is right, thus the area is $\sqrt{1^2+3^2}\times\sqrt{1^2+3^2}\times\frac{1}{2}=10\times\frac{1}{2}=5$ thus the fraction of the total is $\frac{5}{30}=\boxed{(\text{A})\frac{1}{6}}$.

By the Shoelace theorem, the area of $\triangle ABC=\left|\frac{1}{2}(15+4+4-1-20-12)\right|=\left|\frac{1}{2}(-10)\right|=5$. This means the fraction of the total area is $\frac{5}{30}=\boxed{(\text{A})\frac{1}{6}}$.

The smallest rectangle that follows the grid lines and completely encloses $\triangle ABC$ has an area of 12, where $\triangle ABC$ splits the rectangle into four triangles. The area of $\triangle ABC$ is theretore $12-\left(\frac{4\cdot 2}{2}+\frac{3\cdot 1}{2}+\frac{3\cdot 1}{2}\right)=12-\left(4+\frac{3}{2}+\frac{3}{2}\right)=12-7=5$. that means that $\triangle ABC$ takes up $\frac{5}{30}=\boxed{(\text{A})\frac{1}{6}}$ of the grid.

Using Pick's Theorem, the area of the triangle is $4+\frac{4}{2}-1=5$. Therefore, the triangle takes up $\frac{5}{30}=\boxed{(\text{A})\frac{1}{6}}$ of the grid.

33 The area of the shaded region $BEDC$ in parallelogram $ABCD$ is (　). (Adapted from 1989 AJHSME Problem, Question #15)



A. 24　　B. 48　　C. 60　　D. 64　　E. 80

Answer D

Solution Let $[ABC]$ denote the area of figure ABC.

Clearly, $[BEDC] = [ABCD] - [ABE]$. Using basic area formulas,

$[ABCD] = (BC)(BE) = 80$

$[ABE] = \dfrac{(BE)(AE)}{2} = 4(AE)$

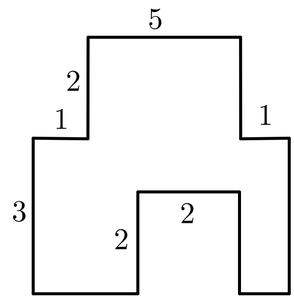
since $AE + ED = BC = 10$ and $ED = 6$, $AE = 4$ and the area of $\triangle ABE$ is $4(4) = 16$.

Finally, we have $[BEDC] = 80 - 16 = 64 \rightarrow \mathrm{D}$

Notice that $BEDC$ is a trapezoid. Therefore its area is

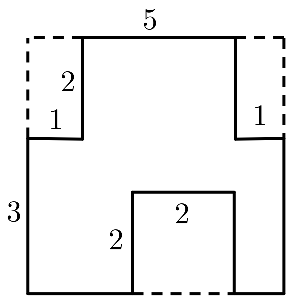
$8\left(\dfrac{6+10}{2}\right) = 8\left(\dfrac{16}{2}\right) = 8(8) = 64 \Rightarrow \mathrm{D}.$

34 Find the area of the figure below.



Answer 27

Solution

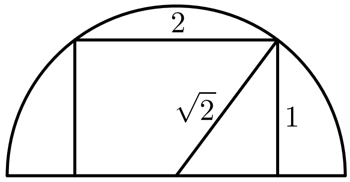


35 A 1×2 rectangle is inscribed in a semicircle with longer side on the diameter. What is the area of the semicircle? (　　) . (2013 AMC 8 Problem, Question #20)

A. $\frac{\pi}{2}$　　B. $\frac{2\pi}{2}$　　C. π　　D. $\frac{4\pi}{3}$　　E. $\frac{5\pi}{3}$

Answer C

Solution



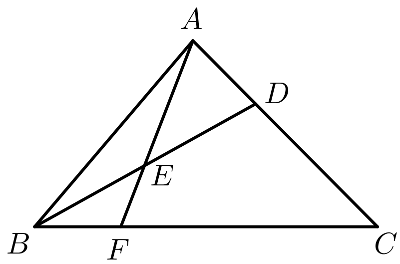
A semicircle has symmetry , so the center is exactly at the midpoint of the 2 side on the rectangle, making the radius, by the Pythagorean Theorem, $\sqrt{1^2+1^2}=\sqrt{2}$. The area is $\sqrt{1^2+1^2}=\sqrt{2}$. The area is $\frac{2\pi}{2}=(\mathrm{C})\pi$.

Double the figure to get a square with side length 2 The circle inscribed around the square has a diameter equal to the diagonal of this square. The diagonal of this square is $\sqrt{2^2+2^2}=\sqrt{8}=2\sqrt{2}$, The circle's radius, therefore, is $\sqrt{2}$

The area of the circle is $\sqrt{2}^2\cdot\pi=2\pi$

Finally, the area of the semicircle is π, so the answer is.

36 In triangle ABC, point D divides side $\overline{AC}$ so that $AD:DC=1:2$. Let E be the midpoint of $\overline{BD}$ and let F be the point of intersection of line BC and line AE. Given that the area of $\triangle ABC$ is 360, what is the area of $\triangle EBF$? (2019 AMC 8 Problem, Question #24)



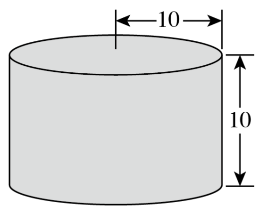
A. 24 B. 30 C. 32 D. 36 E. 40

Answer B

Lesson 4

37 Answer the questions (leave π in the answer):

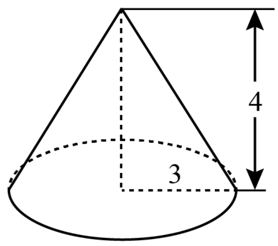
The height of a cylinder is 10, and the radius of the circular base is also 10.



The volume of the cylinder is ______ .

Answer 1000π

38 Find the volume of the following cone. (keep π in the answer)



Answer The volume is: $\frac{1}{3} \times \left(3^2 \times \pi \times 4\right) = 12\pi$

39 (1) If the volume of a cylinder is 375π and the height is 15π, the radius is ______ .

(2) If the surface area of a cylinder is 128π, and the radius is 4, the height is ______ .

Answer $1{:}5$

$2{:}12$

40 (1) The cylinder and the cone have the same base and the same volume. If the height of the cylinder is 3 cm, then the height of the cone is ______ cm.

(2) The cylinder and the cone have the same volume and the same height. If the radius of the base of cylinder is 3 cm, then the radius of the base of cone is ______ cm.

Answer $1{:}9$

$2{:}3\sqrt{3}$

41

Alex and Felicia each have cats as pets. Alex buys cat food in cylindrical cans that are 8 cm in diameter and 12 cm high. Felicia buys cat food in cylindrical cans that are 12 cm in diameter and 8 cm high. What is the ratio of the volume one of Alex's cans to the volume one of Felicia's cans?

Answer $2:3$

42 A company sells peanut butter in cylindrical jars. Marketing research suggests that using wider jars will increase sales. If the diameter of the jars is increased by 25% without altering the volume, by what percent must the height be decreased? (2004 AMC 12A Problem, Question #9)

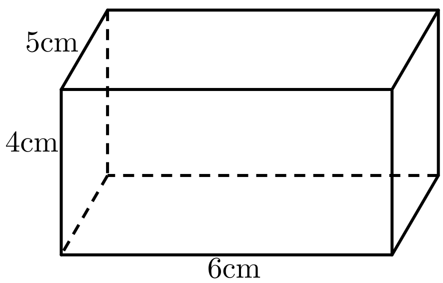
A. 10 B. 25 C. 36 D. 50 E. 60

Answer C

Solution When the diameter is increased by 25%, it is increased by $\frac{5}{4}$, so the area of the base is increased by $\left(\frac{5}{4}\right)^2=\frac{25}{16}$. To keep the volume the same, the height must be $\frac{1}{\frac{25}{16}}=\frac{16}{25}$ of the original height, which is a 36% reduction.

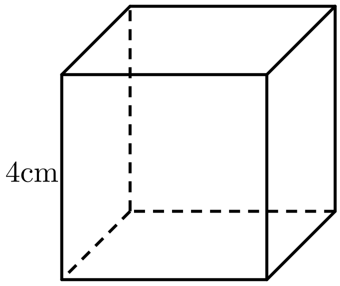
43 Find the volumes of the following solid figures:

(1)



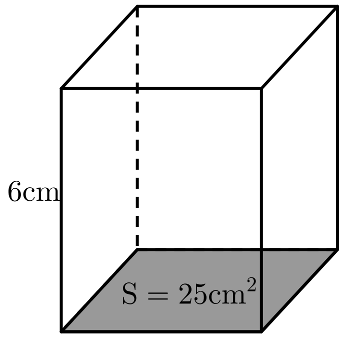
$V=$ ______________ cm^3.

(2)



$V =$ ______________________ cm^3.

(3)



$V =$ ______________________ cm^3.

Answer (1) 120

(2) 64

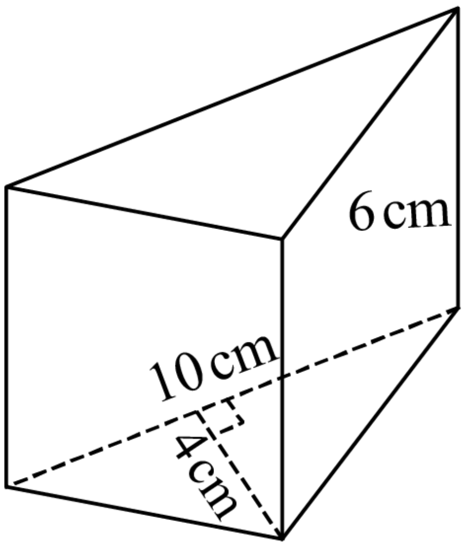
(3) 150

Solution (1) $6 \times 5 \times 4 = 120\left(\mathrm{cm}^3\right)$.

(2) N/A

(3) N/A

44 Find the volume of the following right triangular prism:



Answer 120cm^3

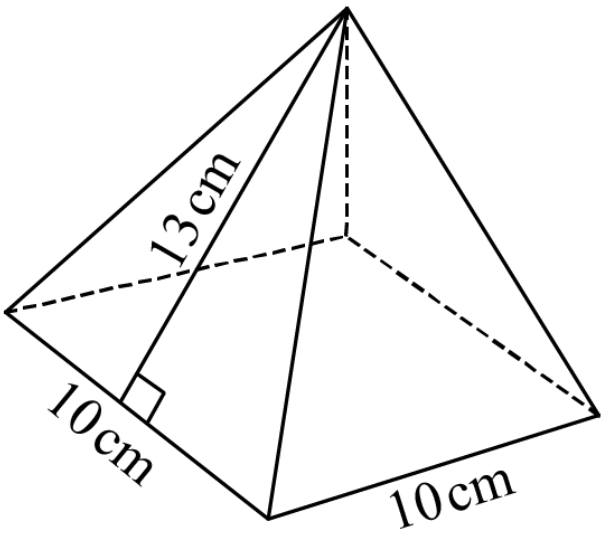
45 For the right square pyramid below:

The area of the base is ______ cm^2.

The area of the four faces is ______ cm^2.

The surface area is ______ cm^2.

The volume is ______ cm^3.



Answer 1:100

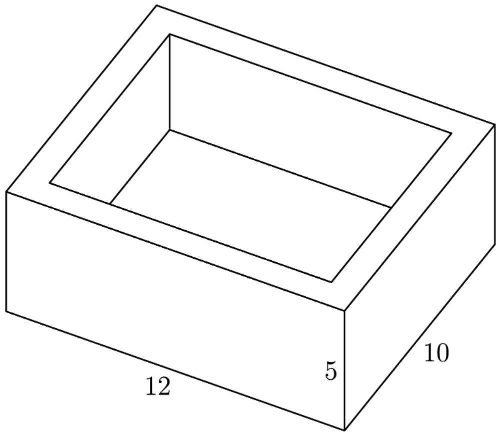
2:260

3:360

4:400

Solution N/A

46 Isabella uses one-foot cubical blocks to build a rectangular fort that is 12 feet long, 10 feet wide, and 5 feet high. The floor and the four walls are all one foot thick. How many blocks does the fort contain? (　　) . (2018 AMC 8 Problems, Question #18)



A. 204　　B. 280　　C. 320　　D. 340　　E. 600

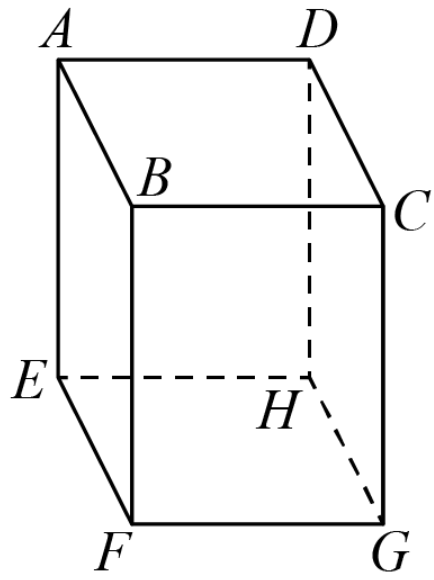
Answer B

Solution There are $10 \cdot 12 = 120$ cubes on the base of the box. Then, for each of the 4 layers above the bottom (as since each cube is 1 foot by 1 foot by 1 foot and the box is 5 feet tall, there are 4 feet left), there are $9 + 11 + 9 + 11 = 40$ cubes. Hence, the answer is $120 + 4 \cdot 40 = \boxed{(\mathrm{B})280}$.

We can just calculate the volume of the prism that was cut out of the original $12 \times 10 \times 5$ box. Each interior side of the fort will be 2 feet shorter than each side of the outside. Since the floor is 1 foot, the height will be 4 feet. So the volume of the interior box is $10 \times 8 \times 4 = 320\mathrm{ft}^3$.

The volume of the original box is $12 \times 10 \times 5 = 600\mathrm{ft}^3$. Therefore, the number of blocks contained in the fort is $600 - 320 = \boxed{(\mathrm{B})280}$.

47 In following rectangular prism, $AB=4, BC=3, BF=6$. Find the volume of triangular prism $ABC-EFG$ and triangular pyramid $A-EFG$ and the length of AG.



Answer $36, 12, \sqrt{61}$