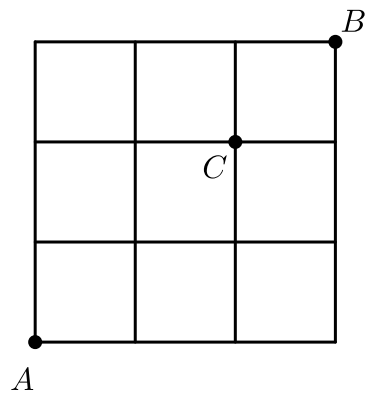


AMC 8 Daily Practice - Nov

Day 1 - Labeling Method

- 1 (1分) An ant starts from point A and moves to point B along the segments. It can only move up or to the right, and it cannot pass through point C. How many different shortest paths are there?



A. 5

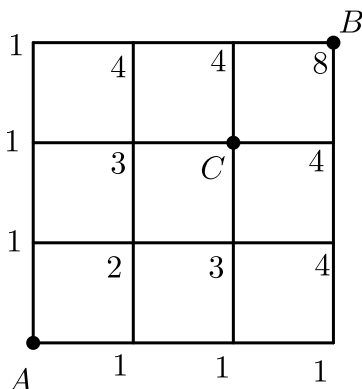
B. 6

C. 7

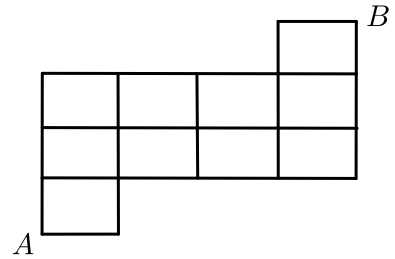
D. 8

Answer D

Solution Place a 1 at A. For each grid intersection, its label equals the sum of the labels of its left and lower neighbors (shortest-step predecessors). Since the ant cannot pass through C, set the label at C to 0 and do not propagate paths through it. Continue filling labels across the grid; the two predecessors of B end up labeled 4 and 4, so the label at B is $4+4=8$. Hence there are 8 shortest paths.



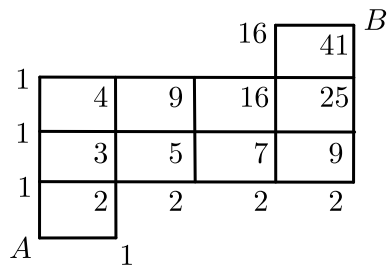
2 (1分) How many different shortest paths are there from A to B ?



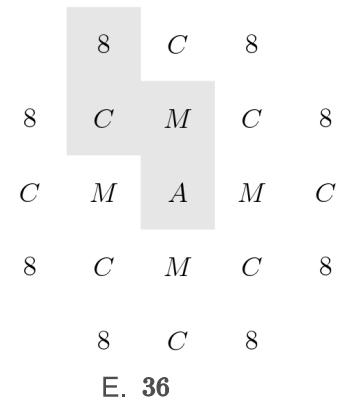
- A. 41 B. 25 C. 16 D. 57 E. 34

Answer A

Solution This problem can be solved using the labeling method. We place a 1 at point A. For each intersection point on the grid, its label equals the sum of the labels of the points that can reach it in one step (here, from the left or from below). We continue filling the grid in this way until we reach B. The point directly left of B has label 25, and the point directly below B has label 16, so the total number of shortest paths to B is $25+16=41$. Thus, there are 41 different shortest paths from A to B.

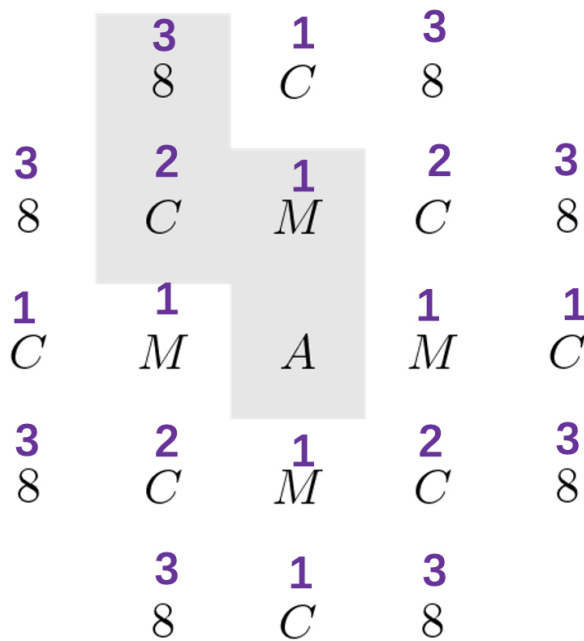


3 (1分) In the arrangement of letters and numerals below, by how many different paths can one spell AMC8? Beginning at the A in the middle, a path allows only moves from one letter to an adjacent (above, below, left, or right, but not diagonal) letter. One example of such a path is traced in the picture. (2017 AMC 8 Problem, Question #15)



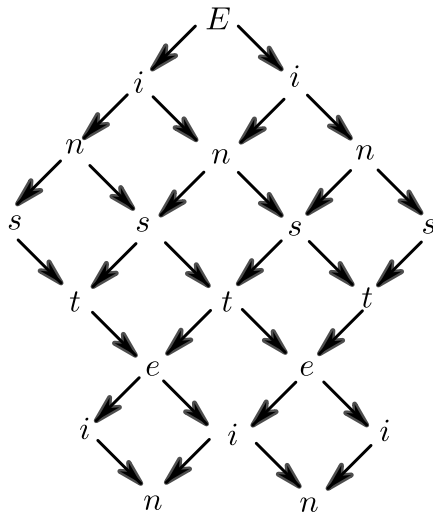
Answer D

Solution $3 \times 8 = 24$



4

(1分) According to the directions indicated by the arrows in the diagram, there are _____ different ways to spell the word "*Einstein*".



A. 30

B. 40

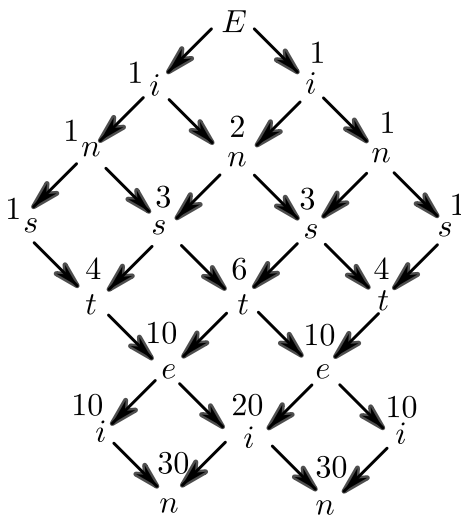
C. 60

D. 80

Answer C

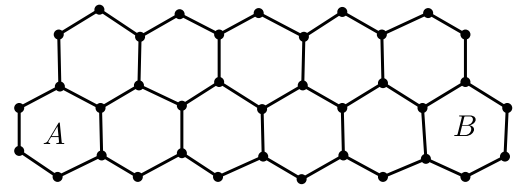
Solution The spelling path goes $E \rightarrow i \rightarrow n \rightarrow s \rightarrow t \rightarrow e \rightarrow i \rightarrow n$ as shown in the diagram.

By applying the labeling method, we know that there are $30 + 30 = 60$ different ways to spell it.



5

(1分) As shown in the figure below, a bee starts from A to go to its home at B . It can only climb from one hive to an adjacent hive moving to the right direction (including upper-right and lower-right) at a time and it is not allowed to go back. How many different routes are there to go home?



A. 55

B. 89

C. 90

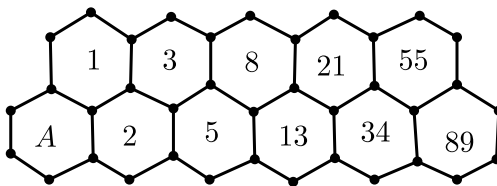
D. 144

Answer B

Solution As the bee "can only climb from one hive to an adjacent hive on the right at a time and it is not allowed to go back", it can only climb from a hive with a smaller number to an adjacent hive with a larger number.

Because the direction that it's heading is clear, we can calculate using the scalar method.

As shown in the figure, there are **89** different ways for the bee to start from **A** to go to its home at **B**.



Day 2 - Logic Reasoning

6 (1分) Teacher Peter gave one red, one white, and one blue balloon to three children, one balloon per child. Based on the statements below, determine which color balloon each child received.

(1) Amy said, "I did not receive the blue balloon."

(2) John said, "I did not receive the white balloon."

(3) Jack said, "I saw that the blue and red balloons were given to the two children above."

Which color balloon did John receive?

A. White

B. Red

C. Blue

Answer C

Solution From Jack's sentence, we know **he cannot have Red or Blue**, so **Jack has White**. Then the remaining colors **Red and Blue** go to **Amy and John**. Amy said she did **not** get Blue, so **Amy has Red**. Therefore, **John has Blue**.

	Red	White	Blue
Amy	✓	×	×
John	×	×	✓
Jack	×	✓	×

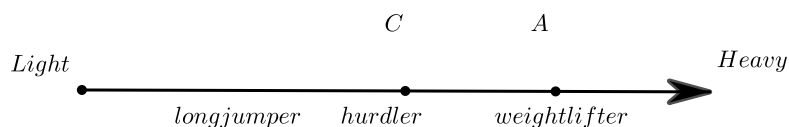
7 (1分) Athletes A, B, and C each won an award in a sports meet. They competed in three different events: weightlifting, hurdles, and long jump, one event per athlete. It is known that Athlete A is heavier than the hurdler, and the long jumper is lighter than both Athlete C and the weightlifter.

The weightlifter is _____, the hurdler is _____, and the long jumper is _____.

- A. A, B, C B. A, C, B C. B, C, A D. C, A, B

Answer B

Solution From the statements "Athlete A is heavier than the hurdler" and "The long jumper is lighter than both Athlete C and the weightlifter," we can determine that B is the long jumper, C is the hurdler, and A is the weightlifter.



8 (1分)

Bridget, Cassie, and Hannah are discussing the results of their last math test. Hannah shows Bridget and Cassie her test, but Bridget and Cassie don't show theirs to anyone. Cassie says, 'I didn't get the lowest score in our class,' and Bridget adds, 'I didn't get the highest score.' What is the ranking of the three girls from highest to lowest? () . (2013 AMC 8 Problems, Question #19)

- A. *Hannah, Cassie, Bridget* B. *Hannah, Bridget, Cassie*
C. *Cassie, Bridget, Hannah* D. *Cassie, Hannah, Bridget*

Answer D

Solution If Hannah did better than Cassie, there would be no way she could know for sure that she didn't get the lowest score in the class. Therefore, Hannah did worse than Cassie. Similarly, if Hannah did worse than Bridget, there is no way Bridget could have known that she didn't get the highest in the class. Therefore, Hannah did better than Bridget, so our order is Cassie, Hannah, Bridget.

9 (1分) John wrote a 4—digit number on a piece of paper and asked Peter to guess it.

Peter asked: "Is the number **1369**?"

John replied: "No, but the number is a multiple of **1369**."

Peter asked again: "Is the number divisible by **9**?"

John replied: "No. You will get a remainder of **4** when the number divided by **9**."

What is the sum of digits of the number written by John?

- A. **22** B. **24** C. **26** D. **28** E. **30**

Answer A

Solution $1369 \div 9$ has a remainder of 1.

$4 \times 1369 \div 9$ has a remainder of 4.

$$4 \times 1369 = 5476$$

10 (1分) Guess a number.

Jack said: "It is a prime number."

Emma said: "It is 9."

Oliver said: "It is an even number."

Sophia said: "It is 15."

The teacher said: "One of Jack and Emma is correct, and one of Oliver and Sophia is also correct."

What is the range of the number?

- A. 1 ~ 4 B. 5 ~ 10 C. 11 ~ 14 D. 15 ~ 20 E. 21 ~ 30

Answer A

Solution If Emma is correct, then neither of Oliver and Sophia can be right.

Thus, Jack and Oliver tell the truth.

The number is 2.

Day 3 - Math in Sports Competition

11 (1分) Five teams compete in a single round-robin tournament, where each pair of teams plays one match. Question: How many matches does each team play, and how many matches are played in total?

- A. 3 , 9 B. 4 , 9 C. 4 , 10 D. 3 , 10

Answer C

Solution Each team has to play against the other 4 teams, so every team needs to play 4 matches. If we count this directly, that would be $4 \times 5 = 20$ matches in total. However, each match is counted twice in this calculation (once for each team), so the actual number of matches is: $20/2 = 10$.

12 (1分) Think Academy organized 8 students to play a badminton tournament in the stadium.

They are paired two-by-two in a knockout format, and a champion needs to be determined among the 8 players. Question: How many matches will be played in total?

- A. 1 B. 2 C. 4 D. 7

Answer D

Solution Method 1: In each round, players are paired two-by-two for knockout matches. With 8 players, we first form $8/2=4$ pairs, so 4 matches are played and 4 players are eliminated. The remaining 4 players form $4/2=2$ pairs, so 2 more matches are needed. Then, 2 players remain and 1 final match is required. Therefore, the total number of matches is:
 $4 + 2 + 1 = 7$.
Method 2: To determine a champion, 7 players must be eliminated. Since each match eliminates exactly one player, the total number of matches is: 7.

13 (1分) Peter, Emma, and Kyler played chess with each other. Peter won 4 games and lost 2 games. Emma won 3 games and lost 3 games. If Kyler lost 3 games, how many games did he win? (2017 AMC 8 Problems, Question #13)

- A. 0 B. 1 C. 2 D. 3 E. 4

Answer B

Solution Given n games, there must be a total of n wins and n losses. Hence, $4+3+K=2+3+3$ where K is Kyler's wins. $K=1$, so our final answer is 1.

14 (1分) Four soccer teams play a single round-robin tournament, where each pair of teams plays one match. If the match ends in a draw, each team receives 1 point; otherwise, the

winning team receives 3 points and the losing team receives 0 points. It is known that the scores of Teams A, B, C, and D are 7 points, 4 points, 4 points, and 1 point respectively.

Question: How many matches ended in a draw?

- A. 2 B. 3 C. 4 D. 6

Answer A

Solution A total of 6 matches are played. If all matches had a winner, the maximum total number of points would be $6 \times 3 = 18$. For each draw, the total points decrease by 1.

So the number of drawn matches is: $18 - (7 + 4 + 4 + 1) = 2$.

15 (1分) Five students A, B, C, D, and E play a tournament in which every pair of students plays one match. In each match, the winner receives 2 points, if the match ends in a draw each player receives 1 point, and the loser receives 0 points. After all matches are finished, the total points of A, B, C, and E are 8, 5, 2, and 2 respectively. Question: How many points did student D score?

- A. 0 B. 1 C. 2 D. 3

Answer D

Solution Since there are 10 matches in total and each match contributes 2 points to the overall score, the total points distributed are 2×10 .

Thus, the score of student D is: $2 \times 10 - 8 - 5 - 2 - 2 = 3$ points.

Day 4 - Inclusion-Exclusion Principle

16 (1分) At a restaurant, 15 people ordered meat dishes and 10 people ordered vegetarian dishes. Furthermore, 5 customers buy both meat dishes and vegetarian dishes, while 8 customers buy neither. The total number of customers is _____ .

A. 15

B. 20

C. 28

D. 35

Answer C

Solution $15 + 10 - 5 = 20$,
 $20 + 8 = 28$.

17 (1分) A survey of Class 1 found that 20 students can swim, 25 students can play basketball, 10 students can do both, and 8 students can do neither. The total number of students in Class 1 is _____ .

A. 38

B. 43

C. 45

D. 47

Answer B

Solution $20 + 25 - 10 + 8 = 43$.

18 (1分) The eighth grade class at Lincoln Middle School has 106 students. Each student takes a math class or a foreign language class or both. There are 65 eighth graders taking a math class, and there are 47 eighth graders taking a foreign language class. How many eighth graders take only a math class and not a foreign language class? (Adapted from 2019 AMC 8 Problems, Question #11)

A. 18

B. 51

C. 65

D. 59

E. 112

Answer D

Solution Since all students must participate at least either. Then besides all in foreign language class, the rest takes exactly only a math class.
 $106 - 47 = 59$.

- 19 (1分) There are 22 students participating in an after-school program offering classes in yoga, bridge, and painting. Each student must take at least one of these three classes, but may take two or all three. There are 12 students taking yoga, 11 taking bridge, and 9 taking painting. There are 7 students taking at least two classes. How many students are taking all three classes?

(Adapted from 2017 AMC 10B Problems, Question #13)

- A. 1 B. 2 C. 3 D. 4 E. 5

Answer C

Solution By PIE (Property of Inclusion/Exclusion), the answer is $12 + 11 + 9 - 7 - 22 = 3$.

- 20 (1分) At Typico High School, 50% of the students like dancing, and the rest actually dislike it. Of those who like dancing, 80% say that they like it, and the rest say that they dislike it. Of those who dislike dancing, 80% say that they dislike it, and the rest say that they like it. What fraction of students who say they dislike dancing actually like it?

- A. 10% B. 15% C. 20% D. 25%

Answer C

Solution $50\% \cdot 20\% = 10\%$ of the people that claim that they dislike dancing actually like it, and $50\% \cdot 80\% = 40\%$. Therefore, the answer is $\frac{10\%}{10\% + 40\%} = \text{(D)}20\%$.

Day 5 - Extremum Principle

- 21 (1分) A rectangular garden has a perimeter of 18 meters. Both the length and the width are positive integers. What is the maximum possible area of the garden, in square meters?

- A. 24 B. 21 C. 20 D. 18

Answer C

Solution Since the sum of the length and width is fixed, the product is largest when the two numbers are as close as possible.

The perimeter is 18 meters, so $\text{length} + \text{width} = 18/2 = 9$

To make the product largest, choose two integers closest to each other: $9 = 4 + 5$.

Thus, the maximum area is $4 \times 5 = 20$ square meters.

22 (1分) Let N be the greatest five—digit number whose digits have a product of **120**. What is the sum of the digits of N ? (2018 AMC 8 Problem, Question #14)

- A. 15 B. 16 C. 17 D. 18 E. 20

Answer D

Solution To make the largest possible five—digit N , you should make the number in the highest digit as large as possible. $120 = 8 \times 5 \times 3 \times 1 \times 1$. So, the sum of digits is $8 + 5 + 3 + 1 + 1 = 18$.

23 (1分) When placing each of the digits **2, 4, 5, 6, 9** in exactly one of the boxes of this subtraction problem, what is the smallest difference that is possible? (1989 AMC 8 Problems, Question #14)

- A. 58 B. 123 C. 149 D. 171 E. 176
- $$\begin{array}{r} \square \quad \square \quad \square \\ - \quad \square \quad \square \\ \hline \end{array}$$

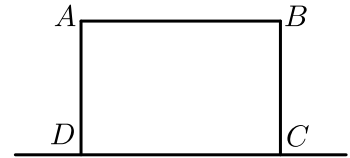
Answer C

Solution

$$\begin{array}{r} \square \quad \square \quad \square \\ - \quad \square \quad \square \\ \hline \end{array}$$

When trying to minimize $a - b$, we minimize a and maximize b . Since in this problem, a is three digit and b is two digit, we set $a = 245$ and $b = 96$. Their difference is **149** $\rightarrow$ **C**.

- 24 (1分) A person plans to use 100 meters of fencing to enclose a rectangular garden outside their house. One side of the garden can be formed by the existing wall and does not require fencing. What is the maximum possible area of the garden, in square meters?

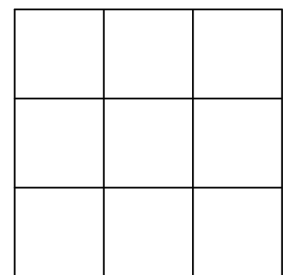


- A. 1240 B. 1250 C. 1260 D. 1270

Answer B

Solution Let the side parallel to the wall be a meters, and the other two sides be b meters each. Then the fencing gives: $a+2b=100$. Since the sum $a+2b$ is fixed, the product $a \cdot 2b=2ab$ reaches its maximum when $a=2b=50$. So, $2ab=50 \times 50=2500$. Thus, the maximum value of the garden area ab is: $2500/2=1250$ square meters.

- 25 (1分) Placing no more than one **X** in each small square, what is the greatest number of **X**'s that can be put on the grid shown without getting three **X**'s in a row vertically, horizontally, or diagonally? (1988 AMC 8 Problems, Question #16)



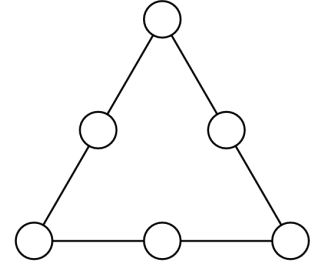
- A. 2 B. 3 C. 4 D. 5 E. 6

Answer E

Solution By the Pigeonhole Principle, if there are at least **7X's**, then there will be some row with **3X's**. We can put in **6** by leaving out the three boxes in one of the main diagonals.

Day 6 - Operational Problems

- 26 (1分) In a magic triangle, each of the six whole numbers 10 – 15 is placed in one of the circles so that the sum, S , of the three numbers on each side of the triangle is the same. The largest possible value for S is ? (1985 AMC 8 Problems, Question #24)



- A. 36 B. 37 C. 38 D. 39 E. 40

Answer D

Solution Let the number in the top circle be a and then b, c, d, e , and f , going in clockwise order.

Then, we have $S = a + b + c$, $S = c + d + e$, $S = e + f + a$,

Adding these equations together, we get

$$\begin{aligned} 3S &= (a + b + c + d + e + f) + (a + c + e) \\ &= 75 + (a + c + e) \end{aligned}$$

where the last step comes from the fact that since a, b, c, d, e , and f are the numbers 10 – 15 in some order, their sum is $10 + 11 + 12 + 13 + 14 + 15 = 75$.

The left hand side is divisible by 3 and 75 is divisible by 3, so $a + c + e$ must be divisible by 3

. The largest possible value of $a + c + e$ is then $15 + 14 + 13 = 42$, and the corresponding value of S is $\frac{75 + 42}{3} = 39$, which is choice D.

It turns out this sum is attainable if you let $a = 15, b = 10, c = 14, d = 12, e = 13, f = 11$.

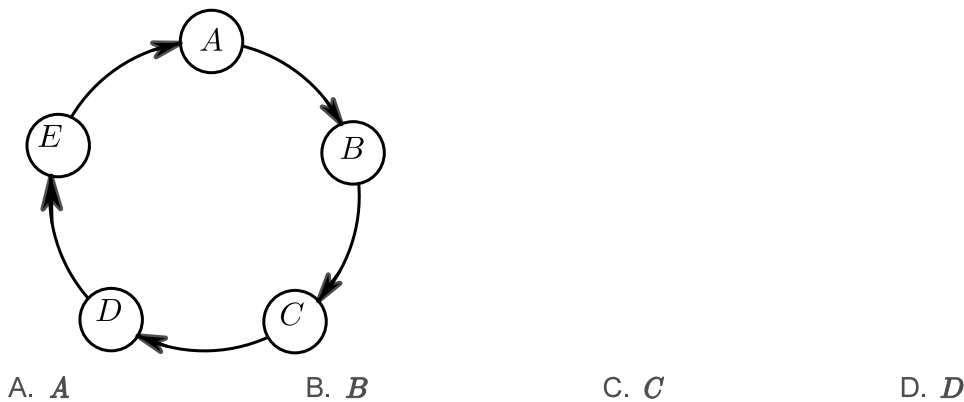
- 27 (1分) There are 2018 students standing in a line, They count off in the repeating pattern 1 , 2 , 3 , 4 , 3 , 2 , 1 , 2 , 3 , 4 , 3 , 2 , 1 ... What number will the 2018th student say?

- A. 1 B. 2 C. 3 D. 4

Answer B

Solution We observe that the counting pattern has a cycle of 6 numbers. $2018 \div 6 = 336$ (cycles)
 $\dots\dots 2$, Therefore, the 2018th student will say 2.

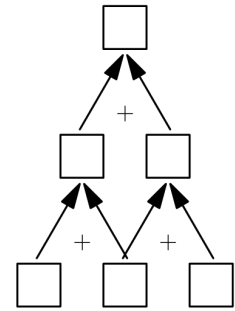
28 (1分) As shown in the diagram, five people (A, B, C, D, and E) stand in a circle playing a passing game. Starting from A, the ball is passed clockwise to the next person each time. After the ball has been passed 17 times, who is holding the ball?



Answer C

Solution We consider the five people (A, B, C, D, E) as forming one repeating sequence of 5 passes.
 $17 \div 5 = 3 \dots\dots 2$. So after 3 complete rounds of passing, there are 2 passes remaining.
 Beginning from A, the first pass goes to B and the second pass to C. Thus, after 17 passes, the ball is held by C.

29 (1分) Three different one-digit positive integers are placed in the bottom row of cells. Numbers in adjacent cells are added and the sum is placed in the cell above them. In the second row, continue the same process to obtain a number in the top cell. What is the difference between the largest and smallest numbers possible in the top cell? (2006 AMC 8 Problems, Question #22)



- A. 16 B. 24 C. 25 D. 26 E. 35

Answer D

Solution If the lower cells contain A , B and C . Then the second row will contain $A + B$ and $B + C$, and the top cell will contain $A + 2B + C$. To obtain the smallest sum, place 1 in the center cell and 2 and 3 in the outer ones. The top number will be 7. For the largest sum, place 9 in the center cell and 7 and 8 in the outer ones. This top number will be 33. The difference is $33 - 7 = 26$.

30 (1分) Lucy is playing an adding-number game in the 4×4 table. In the table, all the grids are starting with 0. For each step, Lucy can choose one of the three operations as below:

- ① Choose one grid and add 4 to the number.
- ② Choose two adjacent grids (with common lines) and add 2 to each of their numbers.
- ③ Choose a 2×2 square and add 1 to each of the numbers inside.

After many operations, Lucy finds that all the numbers are different from each other. How many times of operations at least does she make?

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

- A. 30 B. 45 C. 48 D. 36 E. 24

Answer A

Solution No matter which step Lucy chooses, she can make the sum of numbers adding by 4.

If all numbers are different from each other, she can fill in $0 - 15$ in the 16 grids, which has a sum of $(0 + 15) \times 16 \div 2 = 120$. For each step, she can add 4 , so Lucy needs at least $120 \div 4 = 30$ steps.

After trying, we can get:

1	5	2	6
3	7	10	14
9	13	0	8
11	15	4	12