Think Academy

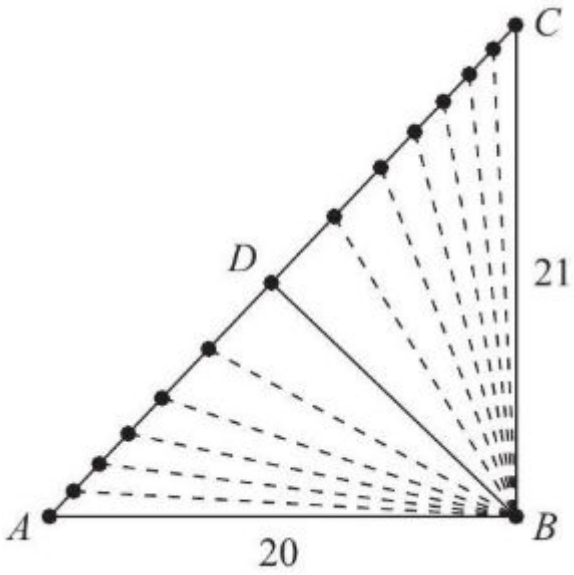
AMC Day18

1 （1分）Right triangle ABC has leg lengths $AB = 20$ and $BC = 21$. Including $\overline{AB}$ and $\overline{BC}$, how many line segments with integer length can be drawn from vertex B to a point on hypotenuse $\overline{AC}$? （2018 AMC 10A Problem 16）

A. 5　　B. 8　　C. 12　　D. 13　　E. 15

Answer D

Solution



Let D be the foot of the altitude from B to $\overline{AC}$. By the Pythagorean Theorem,

$AC = \sqrt{20^2 + 21^2} = 29.$

Computing the area of $\triangle ABC$ in two ways gives $\frac{1}{2} \cdot 20 \cdot 21 = \frac{1}{2} \cdot 29 \cdot BD$,

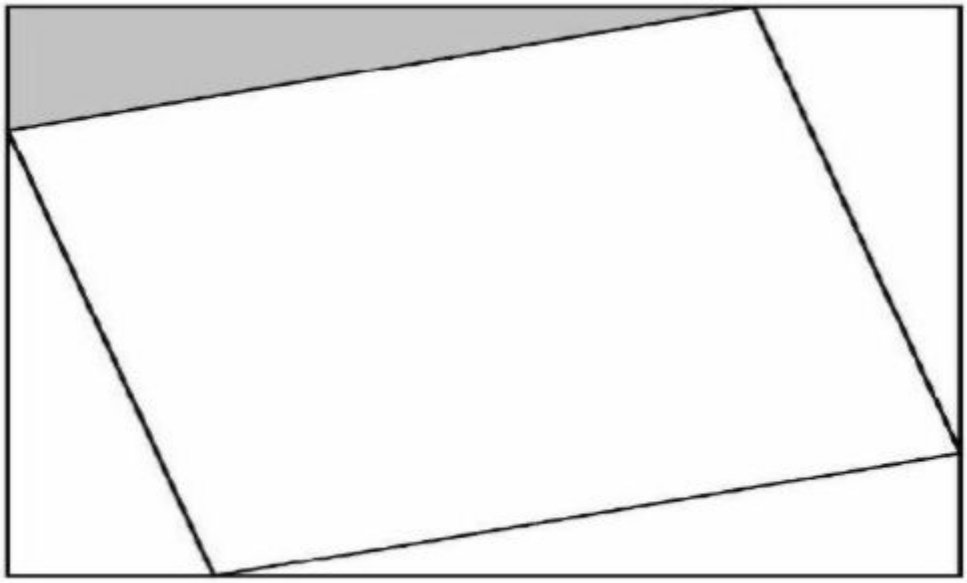
so $BD = \frac{420}{29}$,　　$14 < BD < 15.$

As a point moves from D toward either endpoint of $\overline{AC}$, its distance from B increases. Along $\overline{DA}$, the distance increases from $\frac{420}{29}$ to 20. Along $\overline{DC}$, it increases from $\frac{420}{29}$ to 21.

Thus, each integer length from 15 through 20 occurs twice, once on each side of D, while length 21 occurs only once, at C.

Therefore, the total number of segments is $2(20-15+1)+1=\boxed{(\mathrm{D})\ 13}$.

2 （1分）A square of area 2 is inscribed in a square of area 3, creating four congruent triangles, as shown below. What is the ratio of the shorter leg to the longer leg in the shaded right triangle? （2023 AMC 10A Problem 11）



A. $\frac{1}{5}$

B. $\frac{1}{4}$

C. $2-\sqrt{3}$

D. $\sqrt{3}-\sqrt{2}$

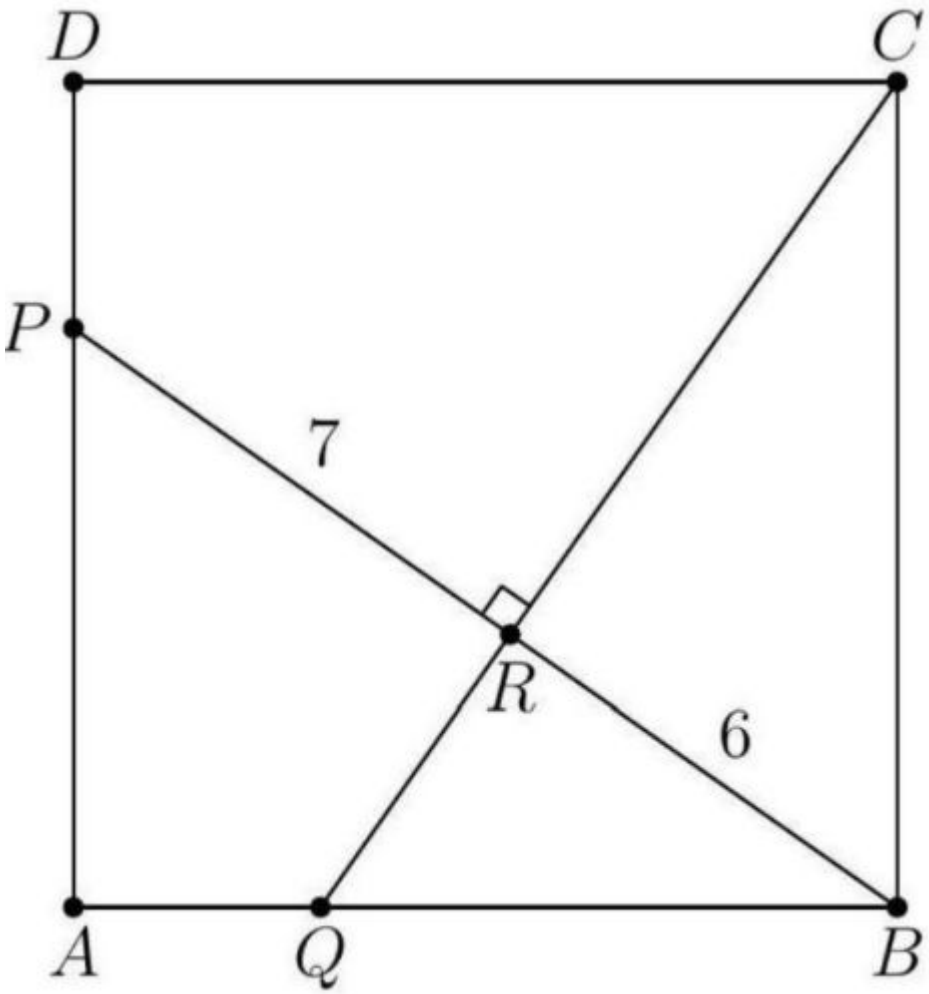
E. $\sqrt{2}-1$

Answer C

Solution Let x be the length of the shorter leg of the right triangle. Then the longer leg has length $\sqrt{3}-x$. The hypotenuse has length $\sqrt{2}$. By the Pythagorean Theorem, $x^2+(\sqrt{3}-x)^2=2$. This equation simplifies to $2x^2-2\sqrt{3}x+1=0$, and the Quadratic Formula gives $x=\frac{\sqrt{3}\pm 1}{2}$. Since x is the shorter leg, we take $x=\frac{\sqrt{3}-1}{2}$, $\sqrt{3}-x=\frac{\sqrt{3}+1}{2}$. The requested ratio is $\frac{\frac{1}{2}(\sqrt{3}-1)}{\frac{1}{2}(\sqrt{3}+1)}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}=\boxed{(\mathrm{C})\ 2-\sqrt{3}}$.

3 （1分）

In square $ABCD$, points P and Q lie on $\overline{AD}$ and $\overline{AB}$, respectively. Segments $\overline{BP}$ and $\overline{CQ}$ intersect at right angles at R, with $BR = 6$ and $PR = 7$. What is the area of the square? (2021 Fall AMC 10B Problem 15)



A. 85 B. 93 C. 100 D. 117 E. 125

Answer D

Solution Note that $\triangle APB \cong \triangle BQC$: both are right triangles, $AB = BC$, and $\angle ABP = \angle BCQ$ because $AB \perp BC$ and $BP \perp CQ$. Thus, $QC = PB = PR + RB = 7 + 6 = 13.$

Let $CR = x$. Then $RQ = 13 - x$.

Since BR is the altitude to the hypotenuse of right triangle QBC, we have $QR \cdot RC = BR^2$.

Substituting gives $(13 - x)x = 36$, so $x^2 - 13x + 36 = 0$, $(x - 4)(x - 9) = 0$.

Since Q lies on $\overline{AB}$, we have $BQ \le AB = BC$.

Applying the Pythagorean Theorem to $\triangle BQR$ and $\triangle BCR$ gives

$$BQ^2 = BR^2 + QR^2, \qquad BC^2 = BR^2 + CR^2.$$

Therefore, $QR \le CR$, so $x = 4$ is impossible. Hence $CR = 9$.

Finally, by the Pythagorean Theorem in $\triangle BCR$, $BC^2 = BR^2 + CR^2 = 6^2 + 9^2 = 117.$ Thus, the area of the square is $\boxed{\textbf{(D) } 117}$.