Think Academy

AMC Day20

1 （1分）Line segment $\overline{AB}$ is a diameter of a circle with $AB = 24$. Point C, not equal to A or B, lies on the circle. As point C moves around the circle, the centroid (center of mass) of $\triangle ABC$ traces out a closed curve missing two points. To the nearest positive integer, what is the area of the region bounded by this curve? （2018 AMC 10B Problem 12）

A. 25　　B. 38　　C. 50　　D. 63　　E. 75

Answer C

Solution Let O be the center of the circle and G be the centroid of $\triangle ABC$.

Since $\overline{AB}$ is a diameter, O is the midpoint of $\overline{AB}$, so $\overline{CO}$ is a median of the triangle.

The centroid divides each median in a $2:1$ ratio measured from the vertex. Therefore, $OG = \frac{1}{3}OC$.

The original circle has radius $OC = \frac{24}{2} = 12$, so $OG = \frac{12}{3} = 4$.

As C moves around the original circle, G traces a circle centered at O with radius 4, missing two points corresponding to $C = A$ and $C = B$. These missing points do not affect the enclosed area.

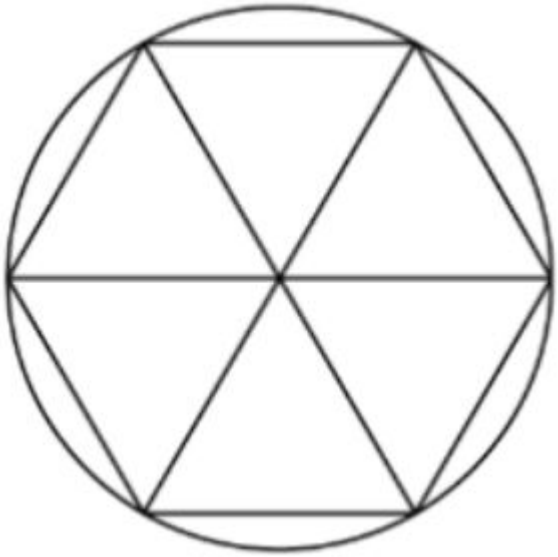
Thus, the area is $\pi \cdot 4^2 = 16\pi \approx 50.27$. To the nearest positive integer, the answer is $\boxed{(C)\ 50}$.

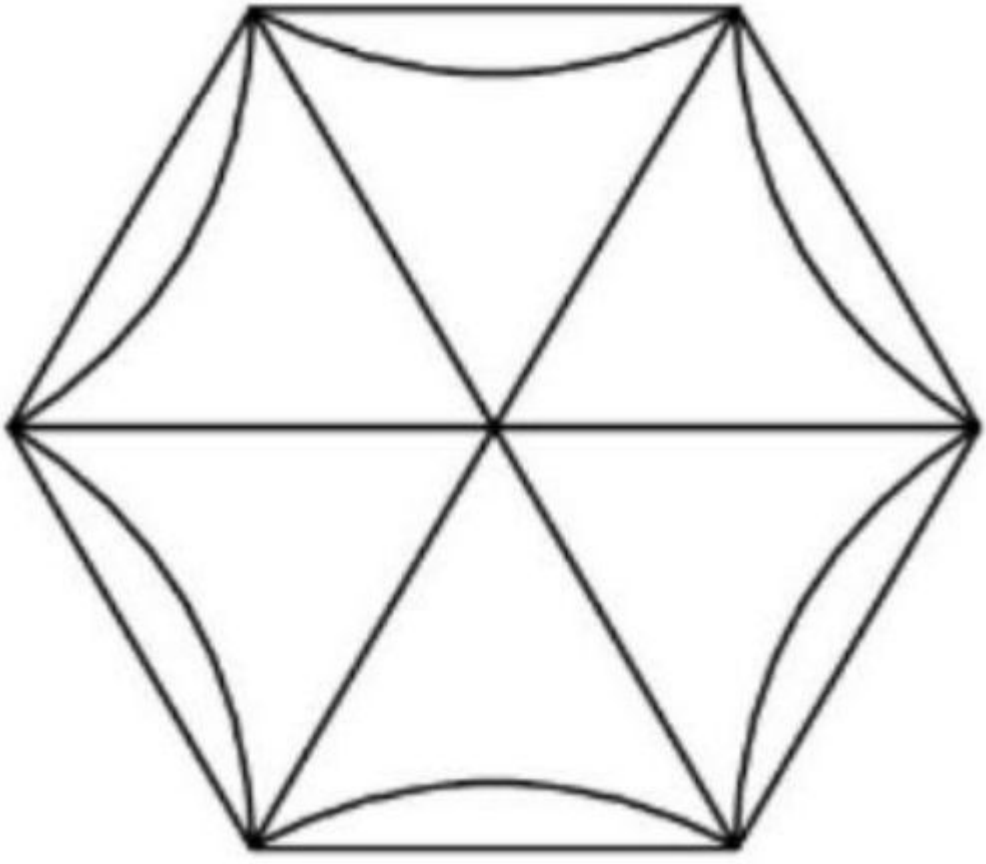
2 （1分）A regular hexagon of side length 1 is inscribed in a circle. Each minor arc of the circle determined by a side of the hexagon is reflected over that side. What is the area of the region bounded by these 6 reflected arcs? （2021 AMC 10B Problem 11）

A. $\frac{5\sqrt{3}}{2} - \pi$　　B. $3\sqrt{3} - \pi$　　C. $4\sqrt{3} - \frac{3\pi}{2}$　　D. $\pi - \frac{\sqrt{3}}{2}$　　E. $\frac{\pi + \sqrt{3}}{2}$

Answer B

Solution



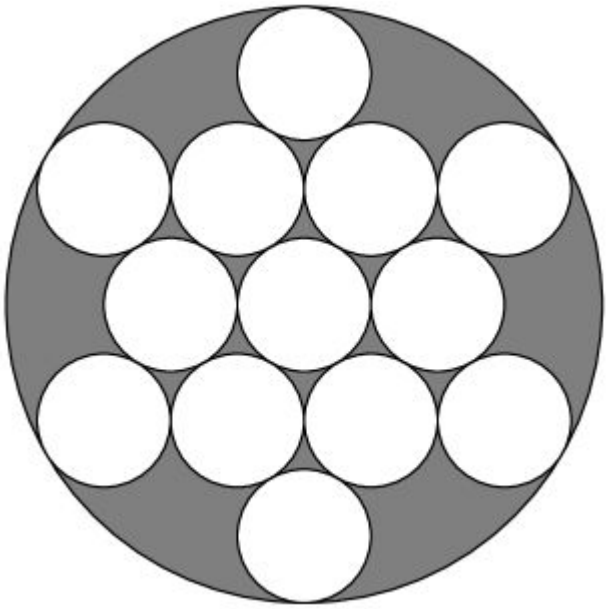


Connecting the center of the circle to the vertices of the regular hexagon divides it into 6 equilateral triangles of side length 1. Thus, the circle has radius 1, and the area of the hexagon is $6 \cdot \frac{\sqrt{3}}{4} = \frac{3\sqrt{3}}{2}$.

Each minor arc and its corresponding side enclose a circular segment. Its area is the area of a $60°$ sector minus the area of an equilateral triangle: $\frac{\pi}{6}-\frac{\sqrt{3}}{4}$. Reflection preserves area, so the total area of the 6 reflected circular segments is $6\left(\frac{\pi}{6}-\frac{\sqrt{3}}{4}\right)=\pi-\frac{3\sqrt{3}}{2}$.

The required region is the hexagon with these 6 segments removed. Therefore, its area is $\frac{3\sqrt{3}}{2}-\left(\pi-\frac{3\sqrt{3}}{2}\right)=\boxed{(\mathrm{B})\ 3\sqrt{3}-\pi}$.

3 （1分）The figure below shows 13 circles of radius 1 within a larger circle. All the intersections occur at points of tangency. What is the area of the region, shaded in the figure, inside the larger circle but outside all the circles of radius 1? （2019 AMC 10A Problem 16）



A. $4\pi\sqrt{3}$

B. 7π

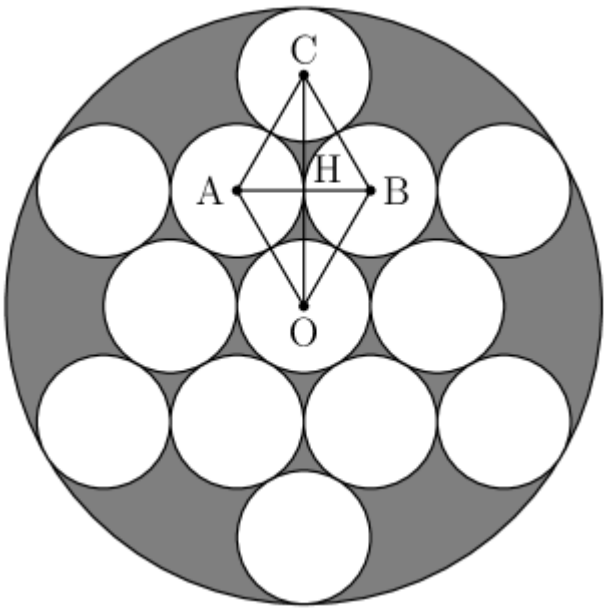
C. $\pi(3\sqrt{3}+2)$

D. $10\pi(\sqrt{3}-1)$

E. $\pi(\sqrt{3}+6)$

Answer A

Solution



In the diagram above, notice that $\triangle OAB$ and $\triangle ABC$ are congruent equilateral triangles with side length 2. Let H be the intersection of $\overline{AB}$ and $\overline{OC}$.

Using the Pythagorean Theorem in $\triangle OHA$, we obtain $OH = \sqrt{OA^2 - AH^2} = \sqrt{2^2 - 1^2} = \sqrt{3}$. Thus, $OC = 2OH = 2\sqrt{3}$. Since the top small circle has radius 1 and is internally tangent to the larger circle, the radius of the larger circle is $R = OC + 1 = 2\sqrt{3} + 1$.

The area of the larger circle is $\pi R^2 = \pi(2\sqrt{3} + 1)^2 = (13 + 4\sqrt{3})\pi$. The sum of the areas of the 13 smaller circles is 13π. Therefore, the shaded area is $(13 + 4\sqrt{3})\pi - 13\pi = \boxed{\text{(A) } 4\pi\sqrt{3}}$.