C. $\frac{2}{3}$
D. $\frac{3}{4}$
E. $\frac{4}{5}$

Answer D

Solution Solution 1:

Notice that, the shaded area and the non-shaded area are also similar, with the length ratio $1:k$, and therefore an area ratio $1:k^2$.

Thus the shaded area is $\frac{1}{1+k^2}=64\%$, giving $k=\frac{3}{4}$.

Solution 2:

The total area forms an infinite geometric series:

$1-k^2+k^4-k^6+k^8-k^{10}+\cdots=\frac{1}{1-(-k^2)}=\frac{1}{1+k^2}$

Setting $\frac{1}{1+k^2}=0.64$ yields $k=\frac{3}{4}$.

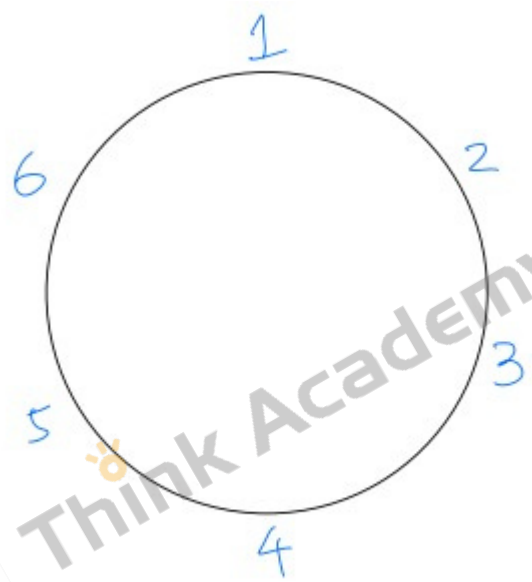
14 Six chairs are arranged around a round table. Two students and two teachers randomly select four of the chairs to sit in. What is the probability that the two students will sit in two adjacent chairs and the two teachers will also sit in two adjacent chairs?

A. $\frac{1}{6}$
B. $\frac{1}{5}$
C. $\frac{2}{9}$
D. $\frac{3}{13}$
E. $\frac{1}{4}$

Answer B

Solution

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We may label the six seats as $1, 2, 3, 4, 5, 6$ on a round table.

Choose the two seats for the two students (order of the two students does not matter).

There are $\binom{6}{2}$ unordered choices in total, while the adjacent pairs are

$(1,2), (2,3), (3,4), (4,5), (5,6), (6,1)$, hence 6 choices. Therefore

$P(\text{adjacent}\ \text{students}) = \frac{6}{\binom{6}{2}} = \frac{2}{5}$

Given that the students are adjacent, without loss of generality, suppose they occupy seats $(1,2)$. The remaining seats are $\{3,4,5,6\}$. Among these four, the adjacent pairs (on the circle) are $(3,4), (4,5), (5,6)$, so there are 3 favorable pairs for the two teachers out of $\binom{4}{2}$ unordered choices. Thus $P(\text{adjacent}\ \text{teachers} \mid \text{adjacent}\ \text{students}) = \frac{3}{\binom{4}{2}} = \frac{1}{2}$

Thus the final answer is

$P(\text{adjacent}\ \text{students}) \cdot P(\text{adjacent}\ \text{teachers} \mid \text{adjacent}\ \text{students}) = \frac{2}{5} \cdot \frac{1}{2} = \frac{1}{5}$

15 In the figure below, $ABEF$ is a rectangle, $\overline{AD} \perp \overline{DE}$, $AF = 7$, $AB = 1$, and $AD = 5$