By Pythagorean theorem, we can see that $AE = 5\sqrt{2}$, $DE = 5$. So $\triangle ADE$ is a right isoceles triangle, $\angle EAD = 45°$.

Make altitude $CG \perp AE$ at G. Set $CG = AG = x$, then $EG = 5\sqrt{2} - x$. As $\triangle ECG \sim \triangle EAB$, $\frac{EG}{GC} = \frac{5\sqrt{2} - x}{x} = \frac{EB}{BA} = 7$

So $x = \frac{5\sqrt{2}}{8}$. Area calculation: $S_{\triangle ABC} = S_{\triangle ABE} - S_{\triangle ACE} = \frac{1}{2} AB \cdot BE - \frac{1}{2} AE \cdot CG = \frac{3}{8}$.

16 There are three jars. Each of three coins is placed in one of the three jars, chosen at random and independently of the placements of the other coins. What is the expected number of coins in a jar with the most coins?

A. $\frac{4}{3}$
B. $\frac{13}{9}$
C. $\frac{5}{3}$
D. $\frac{17}{9}$
E. 2

Answer D

Solution Total arrangements for placing 3 coins in 3 jars: $3^3 = 27$.

If maximum coins per jar = 1: Each jar contains exactly one coin. Count: $3! = 6$.

If maximum coins per jar = 3: One jar holds all coins. Count: 3.

Remaining arrangements $(27 - 6 - 3 = 18)$: One jar has 2 coins, another has 1.

Expected maximum: $\frac{6}{27} \times 1 + \frac{18}{27} \times 2 + \frac{3}{27} \times 3 = \frac{17}{9}$

17 Let N be the unique positive integer such that dividing 273436 by N leaves a remainder of 16 and dividing 272760 by N leaves a remainder of 15. What is the tens digit of N?

A. 0
B. 1
C. 2
D. 3
E. 4

Answer E

Solution Since N is unique, we know that $N = \gcd(273436 - 16, 272760 - 15) = \gcd(273420, 272745)$

Therefore, $N \mid (273420 - 272745) = 675 = 5^2 \times 3^3$.

Think Academy US Online

Since $N \mid \gcd(272745, 675)$ and noting $25 \nmid 272745, 27 \nmid 272745$, we deduce $N \mid 5 \times 3^2 = 45$.

As $45 \mid 272745, 675$, we have $N = 45$.

18 The harmonic mean of a collection of numbers is the reciprocal of the arithmetic mean of the reciprocals of the numbers in the collection. For example, the harmonic mean of 4, 4, and 5 is

$$\frac{1}{\frac{1}{3}\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{5}\right)} = \frac{30}{7}.$$

What is the harmonic mean of all the real roots of the 4050th degree polynomial

$$\prod_{k=1}^{2025}(kx^2-4x-3) = (x^2-4x-3)(2x^2-4x-3)(3x^2-4x-3)\cdots(2025x^2-4x-3)?$$

A. $-\frac{5}{3}$

B. $-\frac{3}{2}$

C. $-\frac{6}{5}$

D. $-\frac{5}{6}$

E. $-\frac{2}{3}$

Answer B

Solution Let $x_k^{(1)}$ and $x_k^{(2)}$ denote the roots of kx^2-4x-3.

By Vieta's formulas: $\frac{1}{x_k^{(1)}}+\frac{1}{x_k^{(2)}} = \frac{x_k^{(1)}+x_k^{(2)}}{x_k^{(1)}\cdot x_k^{(2)}} = \frac{\frac{4}{k}}{-\frac{3}{k}} = -\frac{4}{3}$

Summing over all values: $\sum_{k=1}^{2025}\left(\frac{1}{x_k^{(1)}}+\frac{1}{x_k^{(2)}}\right) = -\frac{4}{3}\times 2025$

Computing the required expression: $\frac{4050}{\sum_{k=1}^{2025}\left(\frac{1}{x_k^{(1)}}+\frac{1}{x_k^{(2)}}\right)} = \frac{4050}{-\frac{4}{3}\times 2025} = -\frac{3}{2}.$

19 An array of numbers is constructed beginning with the numbers $-1\ 3\ 1$ in the top row. Each adjacent pair of numbers is summed to produce a number in the next row. Each row will begin and end with the numbers -1 and 1, respectively. The first three rows are shown below.

$$\begin{array}{ccccccc} & & -1 & & 3 & & 1 \\ & -1 & & 2 & & 4 & & 1 \\ -1 & & 1 & & 6 & & 5 & & 1 \end{array}$$

Think Academy US Online