If the process continues, one of the rows will sum to $12,288$. In that row, what is the third number from the left?

A. -29 B. -21 C. -14 D. -8 E. -3

Answer A

Solution Let S_n represent the sum of row n. Since all the numbers in row $n+1$ can be seen as the sum of the two numbers above it (one number above for the leftmost -1 and rightmost 1), we have the recurrence relation $S_{n+1} = 2S_n$, yielding $S_n = 3 \times 2^{n-1}$. As $12288 = 3 \times 2^{12}$, it is the sum of the 13-th row.

Define a_n as the second element of row n, and b_n as the third element.

From the pattern: $a_{n+1} = a_n - 1$ with $a_1 = 3$, which gives $a_n = 4 - n$ for $n \geq 1$.

With $b_n = b_{n-1} + a_{n-1}$ and $b_1 = 1$:

$$b_n - b_1 = \sum_{k=2}^{n} a_{k-1} = \sum_{k=1}^{n-1} a_k = \sum_{k=1}^{n-1} (4-k) = 4(n-1) - \frac{n(n-1)}{2}$$

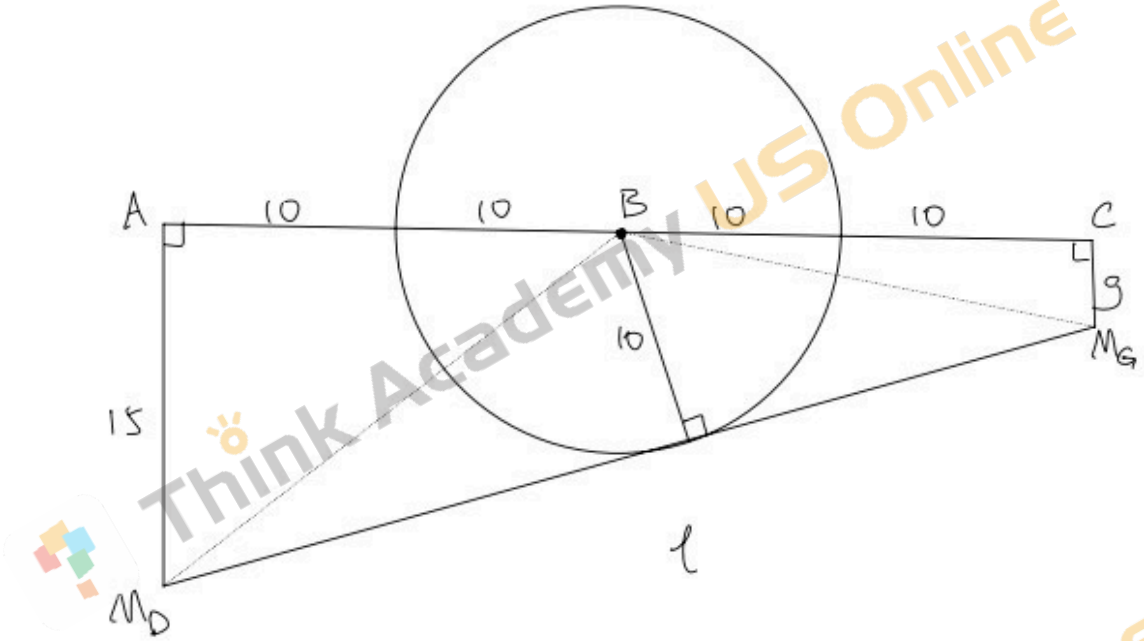
Therefore, $b_n = 4n - 3 - \frac{n(n-1)}{2}$. $b_{13} = 4 \times 13 - 3 - \frac{13 \times 12}{2} = -29$

20 A silo (right circular cylinder) with diameter 20 meters stands in a field. MacDonald is located 20 meters west and 15 meters south of the center of the silo. McGregor is located 20 meters east and $g > 0$ meters south of the center of the silo. The line of sight between MacDonald and McGregor is tangent to the silo. The value of g can be written as $\frac{a\sqrt{b} - c}{d}$, where a, b, c and d are positive integers, b is not divisible by the square of any prime, and d is relatively prime to the greatest common divisor of a and c. What is $a + b + c + d$?

A. 119 B. 120 C. 121 D. 122 E. 123

Answer A

Solution



Let ℓ denote the distance between points M_D and M_G. The trapezoid's area is:

$S_{\triangle BAM_D} + S_{\triangle BCM_G} + S_{\triangle BM_DM_G} = \frac{1}{2}(15 \times 20 + 20 \times 8 + 10 \times \ell)$

Also, trapezoid area = $\frac{1}{2}(15 + g) \times 40$.

Equating: $\frac{1}{2}(300 + 20g + 10\ell) = \frac{1}{2}(15 + g) \times 40$ gives $\ell = 2g + 30$.

Using the Pythagorean theorem: $(2g + 30)^2 = 40^2 + (15 - g)^2$.

Expanding: $4g^2 + 120g + 900 = 1600 + 225 - 30g + g^2$

This simplifies to $3g^2 + 150g - 925 = 0$.

Applying the quadratic formula: $\Delta = 150^2 + 4 \times 3 \times 925 = 33600$

$g = \frac{-150 \pm \sqrt{33600}}{6} = \frac{-75 \pm 20\sqrt{21}}{3}$

Since $g > 0$: $g = \frac{-75 + 20\sqrt{21}}{3}$.

21 A set of numbers is called $sum - free$ if whenever x and y are (not necessarily distinct) elements of the set, $x + y$ is not an element of the set. For example, $\{1, 4, 6\}$ and the empty set are sum-free, but $\{2, 4, 5\}$ is not. What is the greatest possible number of elements in a sum-free subset of $\{1, 2, 3, \cdots, 20\}$?

A. 8 B. 9 C. 10 D. 11 E. 12

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