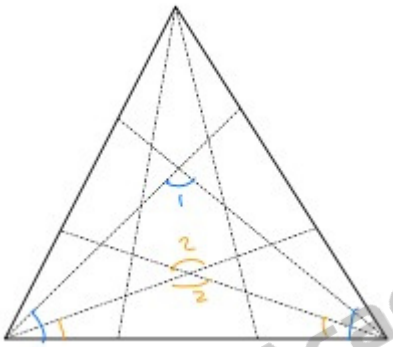


Computing the angles: $\angle 1 = 180^\circ - 40^\circ - 40^\circ = 100^\circ$ $\angle 2 = 180^\circ - 20^\circ - 20^\circ = 140^\circ$

By symmetry, the hexagon's six angles are $100^\circ, 100^\circ, 100^\circ, 140^\circ, 140^\circ, 140^\circ$.

**7** Suppose $a$ and $b$ are real numbers. When the polynomial $x^3 + x^2 + ax + b$ is divided by $x - 1$, the remainder is $4$. When the polynomial is divided by $x - 2$, the remainder is $6$. What is $b - a$?

A. $14$ B. $15$ C. $16$ D. $17$ E. $18$

**Answer** E

**Solution** Define $f(x) = x^3 + x^2 + ax + b$.

Using the given conditions: - $f(1) = 1 + 1 + a + b = 4$ - $f(2) = 8 + 4 + 2a + b = 6$

This gives us: - $a + b = 2$ - $2a + b = -6$

Solving: $a = -8$ and $b = 10$.

Consequently, $b - a = 10 - (-8) = 18$.

**8** Agnes writes the following four statements on a blank piece of paper.

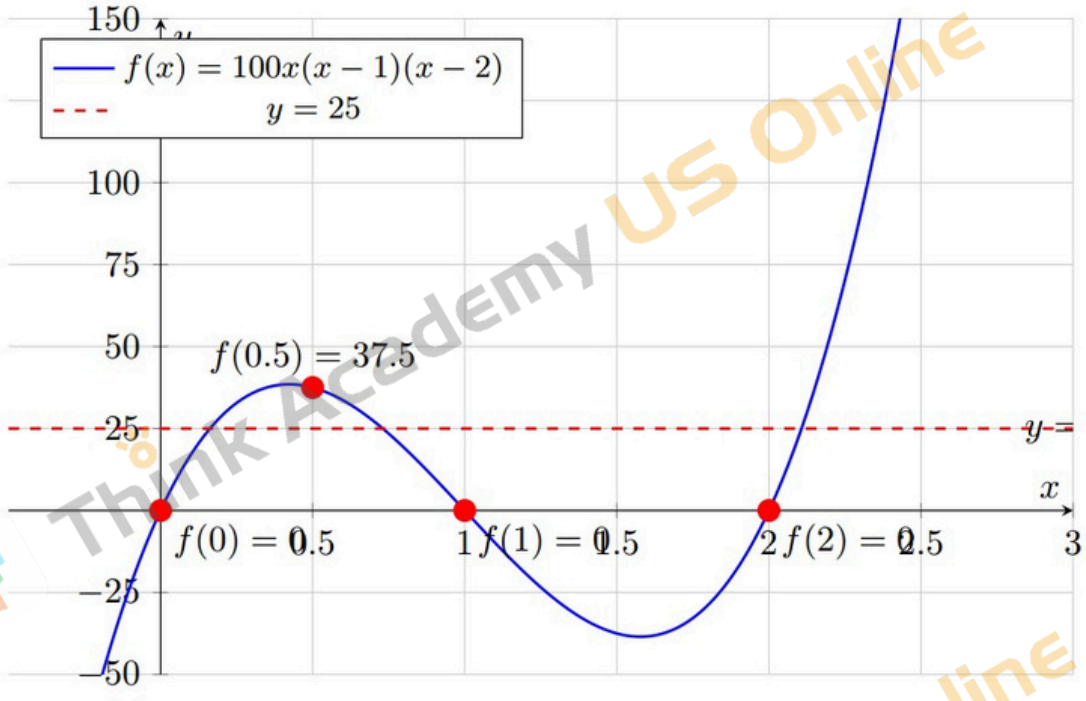
· At least one of these statements is true.

· At least two of these statements are true.

· At least two of these statements are false.

· At least one of these statements is false.

Think Academy US Online




We seek the number of solutions of $f(1-a)=25$, which is equivalent to the number of solutions of $f(x)=25$.

Since $f(x)=100\times x(x-1)(x-2)$, we have $f(0)=f(1)=f(2)=0$. As the coefficient of $x^3$ is positive, we can draw the diagram of $f(x)$.

Note that $f\left(\frac{1}{2}\right)=100\times\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)=25\times\frac{3}{2}>25$

By the intermediate value theorem, there exist $x_1\in\left(0,\frac{1}{2}\right)$ and $x_2\in\left(\frac{1}{2},1\right)$ satisfying $f(x_1)=f(x_2)=25$.

Since $f(2)=0$ and $f(3)>100>25$, there exists $x_3\in(2,3)$ with $f(x_3)=25$.

As $f(x)=25$ is a cubic equation with at most 3 real roots, $x_1,x_2,x_3$ are all the solutions.

10 A semicircle has diameter $\overline{AB}$ and chord $\overline{CD}$ of length 16 parallel to $\overline{AB}$. A smaller semicircle with diameter on $\overline{AB}$ and tangent to $\overline{CD}$ is cut from the larger semicircle, as shown below.

