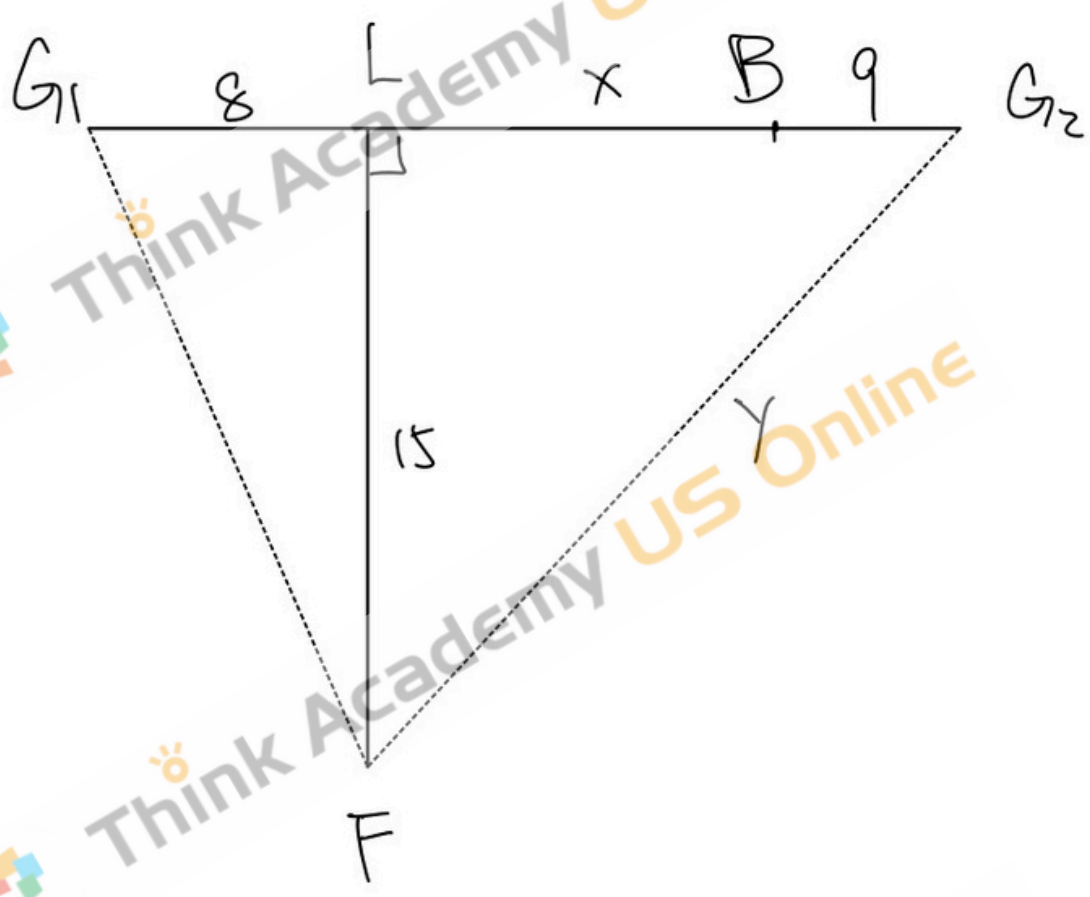
E. $3\frac{6}{7}$

Answer C

Solution



Let $G_2F = y$. Here, $FL \perp G_1G_2$, $BL = x$, $BG_2 = 9$, $LG_1 = 8$, $LF = 15$.

Then $G_1F = \sqrt{LF^2 + LG_1^2} = \sqrt{8^2 + 15^2} = 17$

$BG_1 + FG_1 = 8 + x + 17 = 25 + x$

Also, $9 + y = 25 + x$, which gives $y = 16 + x$.

Since $y^2 = (x+9)^2 + 15^2$, we have $(16+x)^2 = (x+9)^2 + 15^2$, which gives $x = \frac{25}{7}$.

8 Emmy says to Max, "I ordered 36 math club sweatshirts today." Max asks, "How much did each shirt cost?" Emmy responds, "I'll give you a hint. The total cost was $\$ \underline{A}\,\underline{B}\,\underline{B}\,\underline{B}\,\underline{A}$, where A and B are digits and $A \neq 0$." After a pause, Max says, "That was a good price." What is $A + B$?

A. 7 B. 8 C. 11 D. 14 E. 15

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Answer C

Solution Given $36 \mid (ABBBA)_{10}$, we need $9 \mid 3B+2A$ and $4 \mid (BA)_{10}$

From $3 \mid 3B+2A$, we get $3 \mid A$. Since $(BA)_{10}$ is even, A is even, so $2 \mid A$. Thus, $A=6$.

From $9 \mid 3B+2A = 3B+12$, we get $3 \mid B+4$, so $B \in \{2,5,8\}$.

From $4 \mid 10B+A = 10B+6$, we get $2 \mid 5B+3$, which means B is odd.

Therefore $B=5$.

9 How many ordered triples of integers (x,y,z) satisfy the following system of inequalities?

$$-x-y-z \leqslant -2$$

$$-x+y+z \leqslant 2$$

$$x-y+z \leqslant 2$$

$$x+y-z \leqslant 2$$

A. 4 B. 8 C. 11 D. 15 E. 17

Answer C

Solution System of inequalities:

$x+y+z \geq 2$

$-x+y+z \leq 2$

$x-y+z \leq 2$

$x+y-z \leq 2$

Combining the first and second inequalities, we get $x \geq 0$. By symmetry, $x,y,z \geq 0$.

Suppose $0 \leq x \leq y \leq z$ (WLOG).

From the second inequality, $y+z \leq 2+x \leq 2+y$, which gives $z \leq 2$.

Thus, $0 \leq x \leq y \leq z \leq 2$.

Case 1: $x=0$, then from the first and second inequalities, $y+z=2$. Solutions:

$(y,z) = (0,2)$ or $(1,1)$.

Case 2: $x=1$, then from the second inequality, $y+z \leq 3$. Since $1 \leq y \leq z \leq 2$, solutions:

$(y,z) = (1,1)$ or $(1,2)$.

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Case 3: $x = 2$, then $(x, y, z) = (2, 2, 2)$.

In total:

$(1, 1, 1)$ and $(2, 2, 2)$ each give 1 solution

$(0, 0, 2)$, $(0, 1, 1)$, $(1, 1, 2)$ each give 3 solutions

Total number = $1 \times 2 + 3 \times 3 = 11$

10 Let $f(n) = n^3 - 5n^2 + 2n + 8$,and let $g(n) = n^3 - 6n^2 + 5n + 12$. What is the sum of all integer values of n for which $\frac{f(n)}{g(n)}$ is also an integer?

A. 2 B. 3 C. 4 D. 5 E. 6

Answer A

Solution We have the following observations: $\frac{f(n)}{g(n)} - 1 = \frac{f(n) - g(n)}{g(n)} \in \mathbb{Z}$

$f(n) - g(n) = n^2 - 3n - 4 = (n - 4)(n + 1)$

$g(n) = (n + 1)(n - 4)(n - 3)$

$\frac{f(n) - g(n)}{g(n)} = \frac{1}{n - 3}$ when $n \neq -1, 4, 3$

For $\frac{1}{n - 3} \in \mathbb{Z}$, we need $n - 3 = \pm 1$, so $n = 2$ or 4.

However, $n \neq 4$, thus $n = 2$.

11 On Monday, 6 students went to the tutoring center at the same time, and each one was randomly assigned to one of the 6 tutors on duty. On Tuesday, the same 6 students showed up, the same 6 tutors were on duty, and the students were again randomly assigned to the tutors. What is the probability that exactly 2 students met with the same tutor both Monday and Tuesday?

A. $\frac{1}{16}$ B. $\frac{3}{16}$ C. $\frac{1}{4}$ D. $\frac{3}{8}$ E. $\frac{1}{2}$

Answer B