did Ralph buy?

A. 4 B. 5 C. 6 D. 7 E. 8

Answer D

Solution **Solution 1**

So, let there be x pairs of $\$1$ socks, y pairs of $\$3$ socks, and z pairs of $\$4$ socks.

We have $x+y+z=12$, $x+3y+4z=24$, and $x,y,z \geqslant 1$.

Now, we subtract to find $2y+3z=12$, and $y,z \geqslant 1$. It follows that $2y$ is a multiple of 3 and $3z$ is a multiple of 3. Since sum of 2 multiples of 3 = multiple of 3, so we must have $2y=6$.

Therefore, $y=3$, and it follows that $z=2$. Now, $x=12-y-z=12-3-2=\boxed{\textbf{(D) } 7}$, as desired.

Solution 2

Since the total cost of the socks was $\$24$ and Ralph bought 12 pairs, the average cost of each pair of socks is $\frac{\$24}{12}=\$2$.

There are two ways to make packages of socks that average to $\$2$. You can have:

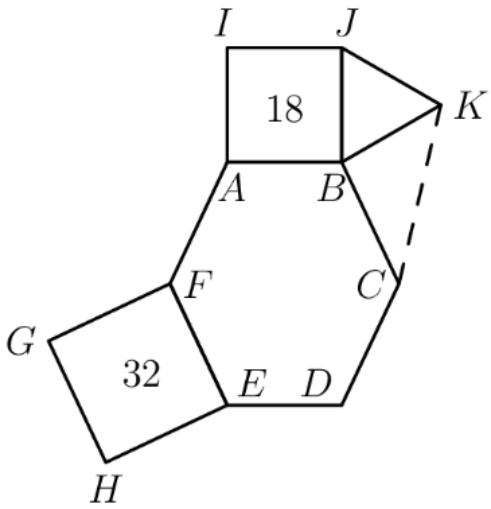
- Two $\$1$ pairs and one $\$4$ pair (package adds up to $\$6$)
- One $\$1$ pair and one $\$3$ pair (package adds up to $\$4$)

Now, we need to solve

$$6a+4b=24,$$

where a is the number of $\$6$ packages and b is the number of $\$4$ packages. We see our only solution (that has at least one of each pair of sock) is $a=2, b=3$, which yields the answer of $2\times 2+3\times 1=\boxed{\textbf{(D) } 7}$.

21 In the given figure, hexagon $ABCDEF$ is equiangular, $ABJI$ and $FEHG$ are squares with areas 18 and 32 respectively, $\triangle JBK$ is equilateral and $FE=BC$. What is the area of $\triangle KBC$?



A. $6\sqrt{2}$ B. 9 C. 12 D. $9\sqrt{2}$ E. 32

Answer C

Solution **Solution 1**

Clearly, since $\overline{FE}$ is a side of a square with area 32, $\overline{FE} = \sqrt{32} = 4\sqrt{2}$. Now, since $\overline{FE} = \overline{BC}$, we have $\overline{BC} = 4\sqrt{2}$.

We know that $\overline{JB}$ is a side of a square with area 18, so $\overline{JB} = \sqrt{18} = 3\sqrt{2}$. Since $\triangle JBK$ is equilateral, $\overline{BK} = \overline{JB} = 3\sqrt{2}$.

Lastly, $\triangle KBC$ is a right triangle. We see that

$\angle JBA + \angle ABC + \angle CBK + \angle KBJ = 360^\circ \rightarrow 90^\circ + 120^\circ + \angle CBK + 60^\circ = 360^\circ \rightarrow \angle CBK, = 90^\circ$

so $\triangle KBC$ is a right triangle with legs $3\sqrt{2}$ and $4\sqrt{2}$. Now, its area is $\frac{3\sqrt{2} \cdot 4\sqrt{2}}{2} = \frac{24}{2} = \boxed{\textbf{(C) } 12}$.

Solution 2

Since $\overline{FE} = \sqrt{32}$, and $\overline{FE} = \overline{BC}$, $\overline{BC} = 4\sqrt{2}$. Meanwhile, $\overline{JB} = 3\sqrt{2}$, and since $\triangle JBK$ is equilateral, $\overline{BK} = 3\sqrt{2}$. If $ABCDEF$ is equiangular, $\angle ABC = \frac{180 \cdot (n-2)}{n} = 120^\circ$, where n is the number of sides of the shape. Adding all the angles around B gives 270°, so $\angle KBC = 360 - 90 = 270^\circ$. Because $\triangle KBC$ is right, the area of $\triangle KBC = \frac{4\sqrt{2} \cdot 3\sqrt{2}}{2} = 12$.

Therefore, the answer is $\boxed{\textbf{(C) } 12}$.