$\therefore 2^2 \cdot 5 \mid n$ and $n \mid 2^2 \cdot 5 \cdot 3^2 = 180$,

$\therefore n = 2^2 \cdot 5 \cdot m$ (m is a factor of 3^2).

$\because gcd(n, 45) = 15$,

$\therefore 3 \cdot 5 \mid n$ and $3^2 \cdot 5 \mid n$,

$\therefore$ If you factorize n, there is only one factor 3,

$\therefore n = 2^2 \cdot 5 \cdot 3 = 60$.

8 A data set consists of 6 (not distinct) positive integers: 1, 7, 5, 2, 5, and X. The average (arithmetic mean) of the 6 numbers equals a value in the data set. What is the sum of all positive values of X?

A. 10 B. 26 C. 32 D. 36 E. 40

Answer D

Solution The possible values for the mean: $5, 7, x$.

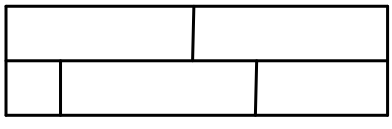
If mean $= 5$, $x = 5 \times 6 - (1 + 2 + 5 + 5 + 7) = 10$;

If mean =7, $x = 7 \times 6 - (1 + 2 + 5 + 5 + 7) = 22$;

If mean =x, $x = 6x - 20$, then $x = 4$.

Sum $= 10 + 22 + 4 = 36$.

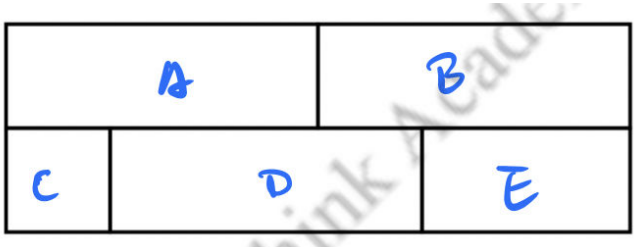
9 A rectangle is partitioned into 5 regions as shown. Each region is to be painted a solid color - red, orange, yellow, blue, or green - so that regions that touch are painted different colors, and colors can be used more than once. How many different colorings are possible?



A. 120 B. 270 C. 360 D. 540 E. 720

Answer D

Solution



D is special as it touches all the other rectangles.

Step 1: Paint D first, we have 5 choices;

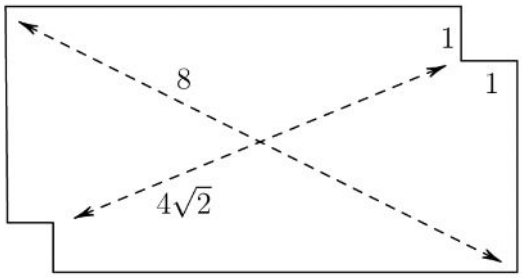
Step 2: Paint C, we have 4 choices;

Step 3: Paint E, we have 5 choices (it's different from D);

Step 4: Paint A and B at last, we have 3 choices for each.

Total number of ways $= 5 \times 4 \times 3 \times 3 \times 3 = 540$.

10 Daniel finds a rectangular index card and measures its diagonal to be 8 centimeters. Daniel then cuts out equal squares of side 1cm at two opposite cormers of the index card and measures the distance between the two closest vertices of these squares to be $4\sqrt{2}$ centimeters, as shown below. What is the area of the original index card?



A. 14 B. $10\sqrt{2}$ C. 16 D. $12\sqrt{2}$ E. 18

Answer E

Solution Assume the length of the index is x and the width is y.

$$x^2 + y^2 = 8^2 = 64$$

$$(x-2)^2 + (y-2)^2 = (4\sqrt{2})^2 = 32$$

$$x^2 - 4x + 4 + y^2 - 4y + 4 = 32$$

$$(x^2 + y^2) - 4(x+y) = 24$$

$$4(x+y) = 40$$