2022 AMC 10B

1 Define $x \diamondsuit y$ to be $|x-y|$ for all real numbers x and y. What's the value of

$$(1 \diamondsuit (2 \diamondsuit 3)) - ((1 \diamondsuit 2) \diamondsuit 3)?$$

A. -2 B. -1 C. 0 D. 1 E. 2

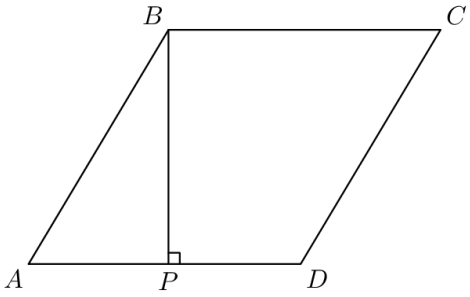
Answer A

Solution Original Expression $= (1 \diamondsuit 1) - (1 \diamondsuit 3)$

$= 0 - 2$

$= 2.$

2 In rhombus $ABCD$, point P lies on segment $\overline{AD}$ such that $BP \perp AD$, $AP = 3$ and $PD = 2$. What is the area of $ABCD$? (Note: The figure is not drawn to scale.)



A. $3\sqrt{5}$ B. 10 C. $6\sqrt{5}$ D. 20 E. 25

Answer D

Solution $AD = AP + PD = 3 + 2 = 5$ $ABCD$ is a rhombus, so $AB = AD = 5$. $\triangle APB$ is a 3-4-5 right triangle, hence $BP = 4$. The area of the rhombus is base times height:

$bh = (AD)(BP) = 5 \cdot 4 = 20.$

3

A donkey suffers an attack of hiccups and the first hiccup happens at $4:00$ one afternoon. Suppose that the donkey hiccups regularly every 5 seconds. At what time does the donkey's 700^{th} hiccup occur?

A. 15 seconds after $4:58$

B. 20 seconds after $4:58$

C. 25 seconds after $4:58$

D. 30 seconds after $4:58$

E. 35 seconds after $4:58$

Answer A

Solution The first hiccup at 4 pm needs to be counted. So the 700^{th} hiccup will occur after $5 \times (700-1) = 3495$ seconds, or 58 minutes and 15 seconds, which will be 15 seconds after $4:58$.

4 How many three-digit positive integers have an odd number of even digits?

A. 150 B. 250 C. 350 D. 450 E. 550

Answer D

Solution There are $5 \times 5 \times 5 \times 2\,(\text{odd/even/odd and odd/odd/even}) + 4 \times 5 \times 5\,(\text{even/odd/odd}) = 350$ ways to let the number have one even digit, while $4 \times 5 \times 5 = 100$ ways to let the number have three even digits. So the answer is $350 + 100 = 450$ such numbers.

5 What is the value of $\dfrac{\left(1+\frac{1}{3}\right)\left(1+\frac{1}{5}\right)\left(1+\frac{1}{7}\right)}{\sqrt{\left(1-\frac{1}{3^2}\right)\left(1-\frac{1}{5^2}\right)\left(1-\frac{1}{7^2}\right)}}$?

A. $\sqrt{3}$ B. 2 C. $\sqrt{15}$ D. 4 E. $\sqrt{105}$

Answer B