Solution Take the difference of squares of the radicand of denominator, we can rewrite the expression in the form of $\frac{a}{\sqrt{ab}}$, where $a = \frac{3}{4} \times \frac{6}{5} \times \frac{8}{7}$ and $b = \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7}$. Here $b = \frac{1}{4}a$. Substitute and simplify to get the result is 2.

6 How many of the first ten numbers of the sequence 121, 11211, 1112111,..... are prime numbers?

A. 0 B. 1 C. 2 D. 3 E. 4

Answer A

Solution Enumerate the terms and find the pattern:

$$\begin{aligned} 121 &= 11 \cdot 11, \\ 11211 &= 101 \cdot 111, \\ 1112111 &= 1001 \cdot 1111, \end{aligned}$$

, and so on. Therefore, no numbers in the sequence would be prime.

7 For how many values of the constant k will the polynomial $x^2 + kx + 36$ have two distinct integer roots?

A. 6 B. 8 C. 9 D. 14 E. 16

Answer B

Solution By Vieta's Formulae, the product of two roots is $36 = 2^2 \times 3^2$, which contains 9 factors. Break down 36 to get $36 = 1 \times 36 = 2 \times 18 = 3 \times 12 = 4 \times 9 = 6 \times 6$, and the signs can be reversed. Since we want distinct roots, the 6×6 case will be neglected, so there are $4 \times 2 = 8$ choices of k.

8 Consider the following 100 sets of 10 elements each:

$$\{1, 2, 3, \ldots, 10, \}$$

$$\{11, 12, 13, ., 20\}$$

$$\{21, 22, 23, \ldots, 30\}$$

:

$$\{991, 992, 993, \ldots, 1000\}.$$

How many of these sets contain exactly two multiples of 7?

A. 40 B. 42 C. 43 D. 49 E. 50

Answer B

Solution Note that for each group, there is at least one multiple of 7 and at most two multiples of 7. There are $\left\lfloor \frac{1000}{7} \right\rfloor = 142$ multiples of 7 under 1000, while there are 100 groups. Therefore, there are $142 - 100 = 42$ sets contain exactly two multiples of 7.

9 The sum $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{2021}{2022!}$ can be expressed as $a - \frac{1}{b!}$, where a and b are positive integers. What is $a + b$?

A. 2020 B. 2021 C. 2022 D. 2023 E. 2024

Answer D

Solution Note that $\frac{n}{(n+1)!} = \frac{1}{n!} - \frac{1}{(n+1)!}$, and therefore this sum is a telescoping sum, which is equivalent to $1 - \frac{1}{2022!}$. Our answer is $1 + 2022 = 2023$.

10 Camila writes down five positive integers. The unique mode of these integers is 2 greater than their median, and the median is 2 greater than their arithmetic mean. What is the least possible value for the mode?

A. 5 B. 7 C. 9 D. 11 E. 13

Answer D

Solution We have $\textbf{mode} = \textbf{median} + 2$ and $\textbf{median} = \textbf{mean} + 2$. If there are three modes, then $\textbf{mode} = \textbf{median}$, which is a contradiction. So there are two modes. Let the five numbers be