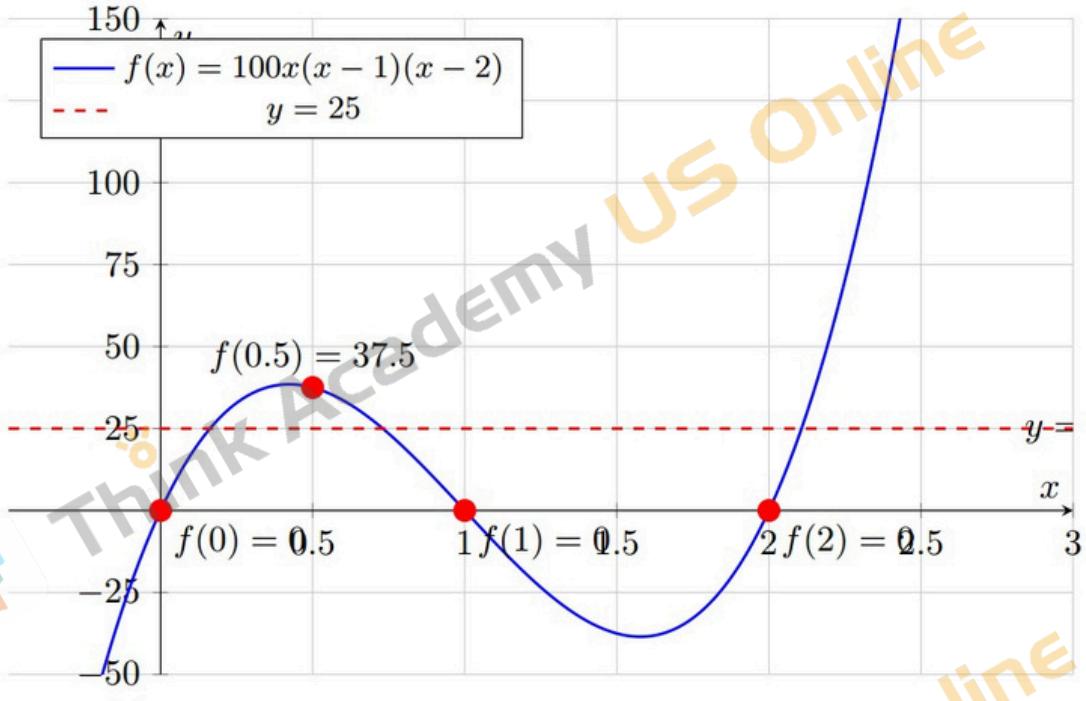


We seek the number of solutions of $f(1-a)=25$, which is equivalent to the number of solutions of $f(x)=25$.

Since $f(x)=100\times x(x-1)(x-2)$, we have $f(0)=f(1)=f(2)=0$. As the coefficient of x^3 is positive, we can draw the diagram of $f(x)$.

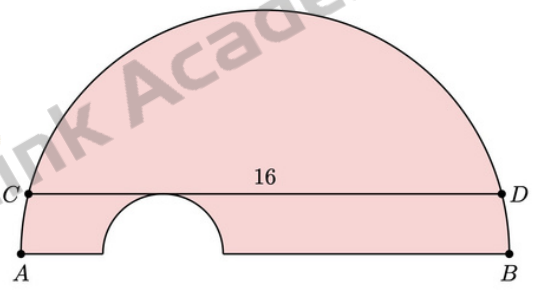
Note that $f\left(\frac{1}{2}\right)=100\times\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)=25\times\frac{3}{2}>25$

By the intermediate value theorem, there exist $x_1\in\left(0,\frac{1}{2}\right)$ and $x_2\in\left(\frac{1}{2},1\right)$ satisfying $f(x_1)=f(x_2)=25$.

Since $f(2)=0$ and $f(3)>100>25$, there exists $x_3\in(2,3)$ with $f(x_3)=25$.

As $f(x)=25$ is a cubic equation with at most 3 real roots, x_1,x_2,x_3 are all the solutions.

10 A semicircle has diameter $\overline{AB}$ and chord $\overline{CD}$ of length 16 parallel to $\overline{AB}$. A smaller semicircle with diameter on $\overline{AB}$ and tangent to $\overline{CD}$ is cut from the larger semicircle, as shown below.

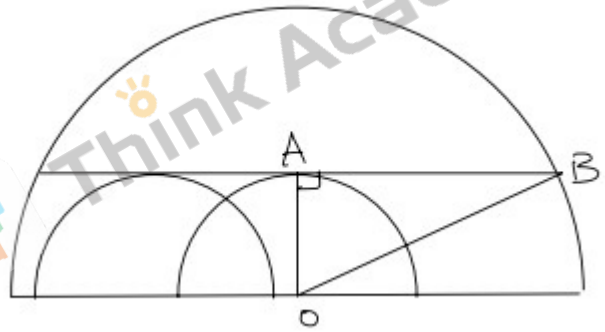


What is the area of the resulting figure, shown shaded?

A. 16π B. 24π C. 32π D. 48π E. 64π

Answer C

Solution



Let r and R denote the radii of the smaller and larger semicircles respectively.

As in the diagram, we can translate the tangent smaller semi-circle to be centered at O. So $OB = R$, $OA = r$, and $AB = 16/2 = 8$. Thus $R^2 - r^2 = 8^2$.

The answer is: $\frac{1}{2}\pi R^2 - \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(R^2 - r^2) = \frac{1}{2}\pi \times 64 = 32\pi$

11 The sequence $1, x, y, z$ is arithmetic. The sequence $1, p, q, z$ is geometric. Both sequences are strictly increasing and contain only integers, and z is as small as possible. What is the value of $x + y + z + p + q$?

A. 66 B. 91 C. 103 D. 132 E. 149

Answer E

Solution Let d be the common difference and r be the common ratio, where $d, r \in \mathbb{N}_+$ with $d, r > 1$.

The equation $1 + 3d = r^3$ holds. Since $r^3 \equiv 1 \pmod 3$, we have $r \equiv 1 \pmod 3$, implying $r \geq 4$.

For $r = 4$: $d = 21$, yielding $x = 22, y = 43, z = 64, p = 4, q = 16$.

$x + y + z + p + q = 22 + 43 + 64 + 4 + 16 = 149$

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