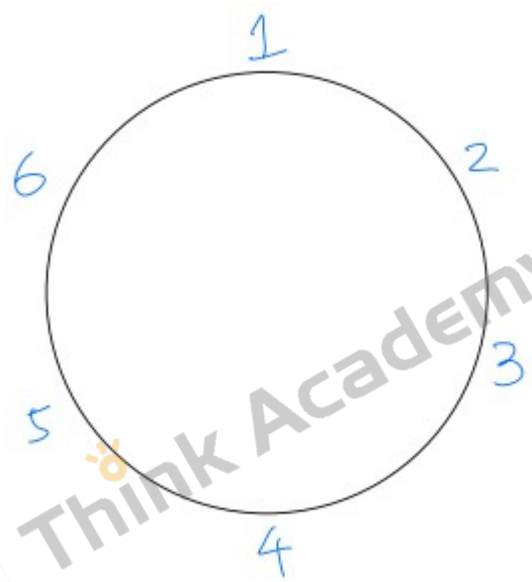


We may label the six seats as $1, 2, 3, 4, 5, 6$ on a round table.

Choose the two seats for the two students (order of the two students does not matter).

There are $\binom{6}{2}$ unordered choices in total, while the adjacent pairs are

$(1,2), (2,3), (3,4), (4,5), (5,6), (6,1)$, hence 6 choices. Therefore

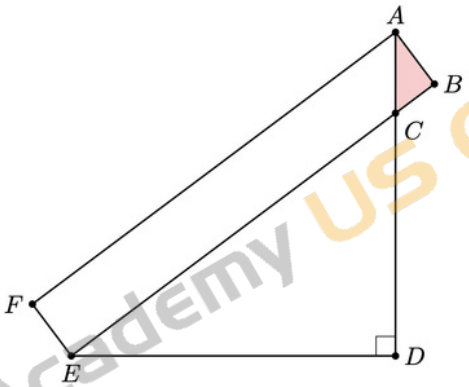
$P(\text{adjacent}\ \text{students}) = \frac{6}{\binom{6}{2}} = \frac{2}{5}$

Given that the students are adjacent, without loss of generality, suppose they occupy seats $(1,2)$. The remaining seats are $\{3,4,5,6\}$. Among these four, the adjacent pairs (on the circle) are $(3,4), (4,5), (5,6)$, so there are 3 favorable pairs for the two teachers out of $\binom{4}{2}$ unordered choices. Thus $P(\text{adjacent}\ \text{teachers} \mid \text{adjacent}\ \text{students}) = \frac{3}{\binom{4}{2}} = \frac{1}{2}$

Thus the final answer is

$P(\text{adjacent}\ \text{students}) \cdot P(\text{adjacent}\ \text{teachers} \mid \text{adjacent}\ \text{students}) = \frac{2}{5} \cdot \frac{1}{2} = \frac{1}{5}$

15 In the figure below, $ABEF$ is a rectangle, $\overline{AD} \perp \overline{DE}$, $AF = 7$, $AB = 1$, and $AD = 5$



What is the area of $\triangle ABC$?

A. $\frac{3}{8}$

B. $\frac{4}{9}$

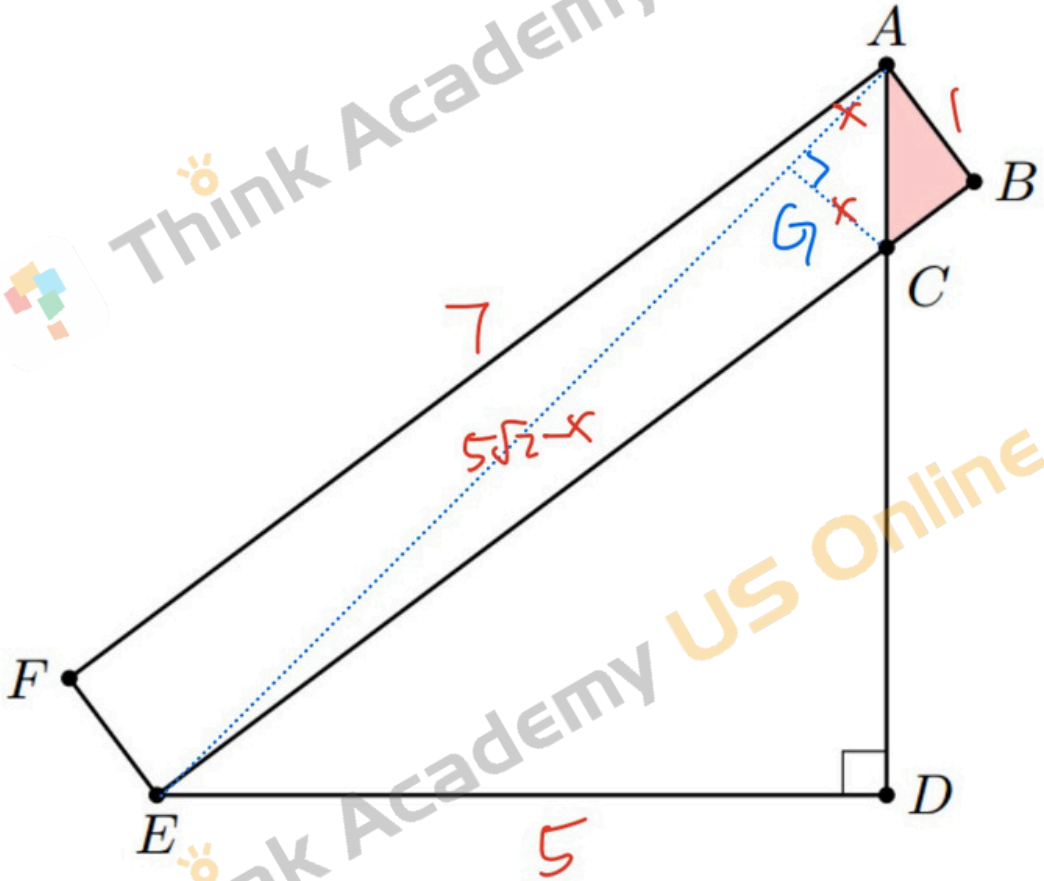
C. $\frac{1}{8}\sqrt{13}$

D. $\frac{7}{15}$

E. $\frac{1}{8}\sqrt{15}$

Answer A

Solution



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By Pythagorean theorem, we can see that $AE = 5\sqrt{2}$, $DE = 5$. So $\triangle ADE$ is a right isoceles triangle, $\angle EAD = 45°$.

Make altitude $CG \perp AE$ at G. Set $CG = AG = x$, then $EG = 5\sqrt{2} - x$. As $\triangle ECG \sim \triangle EAB$, $\frac{EG}{GC} = \frac{5\sqrt{2} - x}{x} = \frac{EB}{BA} = 7$

So $x = \frac{5\sqrt{2}}{8}$. Area calculation: $S_{\triangle ABC} = S_{\triangle ABE} - S_{\triangle ACE} = \frac{1}{2} AB \cdot BE - \frac{1}{2} AE \cdot CG = \frac{3}{8}$.

16 There are three jars. Each of three coins is placed in one of the three jars, chosen at random and independently of the placements of the other coins. What is the expected number of coins in a jar with the most coins?

A. $\frac{4}{3}$
B. $\frac{13}{9}$
C. $\frac{5}{3}$
D. $\frac{17}{9}$
E. 2

Answer D

Solution Total arrangements for placing 3 coins in 3 jars: $3^3 = 27$.

If maximum coins per jar = 1: Each jar contains exactly one coin. Count: $3! = 6$.

If maximum coins per jar = 3: One jar holds all coins. Count: 3.

Remaining arrangements $(27 - 6 - 3 = 18)$: One jar has 2 coins, another has 1.

Expected maximum: $\frac{6}{27} \times 1 + \frac{18}{27} \times 2 + \frac{3}{27} \times 3 = \frac{17}{9}$

17 Let N be the unique positive integer such that dividing 273436 by N leaves a remainder of 16 and dividing 272760 by N leaves a remainder of 15. What is the tens digit of N?

A. 0
B. 1
C. 2
D. 3
E. 4

Answer E

Solution Since N is unique, we know that $N = \gcd(273436 - 16, 272760 - 15) = \gcd(273420, 272745)$

Therefore, $N \mid (273420 - 272745) = 675 = 5^2 \times 3^3$.

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