How many isosceles triangles are there with positive area whose side lengths are all positive integers and whose longest side has length 2025?

A. 2025 B. 2026 C. 3012 D. 3037 E. 4050

Answer D

Solution Consider isosceles triangles with side lengths a, a, b where $a, b \in \mathbb{N}_+$.

Scenario 1: $b = 2025$.

By triangle inequality: $a + a > b$, which gives $2a > 2025$, so $a \geq 1013$.

Therefore, $1013 \leq a \leq 2025$ gives us 1013 valid triangles.

Scenario 2: $a = 2025$.

We have $1 \leq b \leq 2025$, yielding 2025 valid triangles.

Notice that we count the equilateral with side length 2025 twice, so the combined total is $1013 + 2025 - 1 = 3037$ triangles.

4 A team of students is going to compete against a team of teachers in a trivia contest. The total number of students and teachers is 15. Ash, a cousin of one of the students, wants to join the contest. If Ash plays with the students, the average age on that team will increase from 12 to 14. If Ash plays with the teachers, the average age on that team will decrease from 55 to 52. How old is Ash?

A. 28 B. 29 C. 30 D. 32 E. 33

Answer A

Solution Let n denote the number of students, making $(15 - n)$ the number of teachers. Let a represent Ash's age.

From the students' average age: $12n + a = (n + 1) \times 14$

From the teachers' average age: $55 \cdot (15 - n) + a = 52 \cdot (15 - n + 1)$

Solving this system: - From the first equation: $a = 2n + 14$ - From the second equation: $a = 3n + 7$

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Therefore $2n + 14 = 3n + 7$, which gives $n = 7$ and $a = 28$.

5 Consider the sequence of positive integers

$$1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, \cdots$$

What is the 2025th term in this sequence?

A. 5 B. 15 C. 16 D. 44 E. 45

Answer E

Solution Divide the whole sequence at each 1.

The n-th sequence would be: $1, 2, \ldots, n+1, \ldots, 2$, which has length $2n$.

The cumulative length until the end of the n-th sequence is:

$$f(n) = \sum_{k=1}^{n} 2k = 2 \cdot \frac{n(n+1)}{2} = n(n+1)$$

Computing: $f(44) = 44 \times 45 = 1980$ and $f(45) = 45 \times 46 = 2070$.

Hence, the 2025th term appears at position $(2025 - 1980) = 45$ within the 45th sequence $1, 2, \ldots, 46, \ldots, 2$, giving us 45.

6 In an equilateral triangle each interior angle is trisected by a pair of rays. The intersection of the interiors of the middle 20°-angle at each vertex is the interior of a convex hexagon. What is the degree measure of the smallest angle of this hexagon?

A. 80 B. 90 C. 100 D. 110 E. 120

Answer C

Solution

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