Therefore $2n + 14 = 3n + 7$, which gives $n = 7$ and $a = 28$.

5 Consider the sequence of positive integers

$$1, 2, 1, 2, 3, 2, 1, 2, 3, 4, 3, 2, 1, 2, 3, 4, 5, 4, 3, 2, 1, 2, 3, 4, 5, 6, 5, 4, 3, 2, 1, 2, \cdots$$

What is the 2025th term in this sequence?

A. 5 B. 15 C. 16 D. 44 E. 45

Answer E

Solution Divide the whole sequence at each 1.

The n-th sequence would be: $1, 2, \ldots, n+1, \ldots, 2$, which has length $2n$.

The cumulative length until the end of the n-th sequence is:

$$f(n) = \sum_{k=1}^{n} 2k = 2 \cdot \frac{n(n+1)}{2} = n(n+1)$$

Computing: $f(44) = 44 \times 45 = 1980$ and $f(45) = 45 \times 46 = 2070$.

Hence, the 2025th term appears at position $(2025 - 1980) = 45$ within the 45th sequence $1, 2, \ldots, 46, \ldots, 2$, giving us 45.

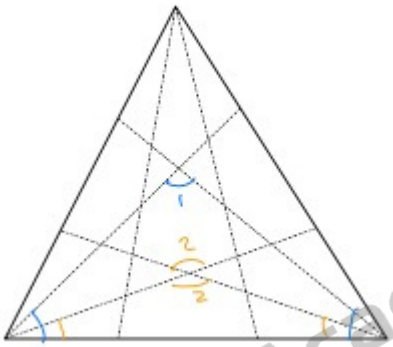
6 In an equilateral triangle each interior angle is trisected by a pair of rays. The intersection of the interiors of the middle 20°-angle at each vertex is the interior of a convex hexagon. What is the degree measure of the smallest angle of this hexagon?

A. 80 B. 90 C. 100 D. 110 E. 120

Answer C

Solution

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Computing the angles: $\angle 1 = 180^\circ - 40^\circ - 40^\circ = 100^\circ$ $\angle 2 = 180^\circ - 20^\circ - 20^\circ = 140^\circ$

By symmetry, the hexagon's six angles are $100^\circ, 100^\circ, 100^\circ, 140^\circ, 140^\circ, 140^\circ$.

7 Suppose a and b are real numbers. When the polynomial $x^3 + x^2 + ax + b$ is divided by $x - 1$, the remainder is 4. When the polynomial is divided by $x - 2$, the remainder is 6. What is $b - a$?

A. 14 B. 15 C. 16 D. 17 E. 18

Answer E

Solution Define $f(x) = x^3 + x^2 + ax + b$.

Using the given conditions: - $f(1) = 1 + 1 + a + b = 4$ - $f(2) = 8 + 4 + 2a + b = 6$

This gives us: - $a + b = 2$ - $2a + b = -6$

Solving: $a = -8$ and $b = 10$.

Consequently, $b - a = 10 - (-8) = 18$.

8 Agnes writes the following four statements on a blank piece of paper.

· At least one of these statements is true.

· At least two of these statements are true.

· At least two of these statements are false.

· At least one of these statements is false.

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