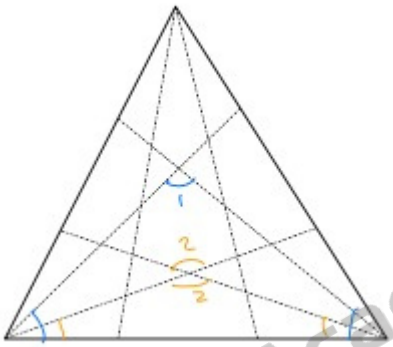


Computing the angles: $\angle 1 = 180^\circ - 40^\circ - 40^\circ = 100^\circ$ $\angle 2 = 180^\circ - 20^\circ - 20^\circ = 140^\circ$

By symmetry, the hexagon's six angles are $100^\circ, 100^\circ, 100^\circ, 140^\circ, 140^\circ, 140^\circ$.

7 Suppose a and b are real numbers. When the polynomial $x^3 + x^2 + ax + b$ is divided by $x - 1$, the remainder is 4. When the polynomial is divided by $x - 2$, the remainder is 6. What is $b - a$?

A. 14 B. 15 C. 16 D. 17 E. 18

Answer E

Solution Define $f(x) = x^3 + x^2 + ax + b$.

Using the given conditions: - $f(1) = 1 + 1 + a + b = 4$ - $f(2) = 8 + 4 + 2a + b = 6$

This gives us: - $a + b = 2$ - $2a + b = -6$

Solving: $a = -8$ and $b = 10$.

Consequently, $b - a = 10 - (-8) = 18$.

8 Agnes writes the following four statements on a blank piece of paper.

· At least one of these statements is true.

· At least two of these statements are true.

· At least two of these statements are false.

· At least one of these statements is false.

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Each statement is either true or false. How many false statements did Agnes write on the paper?

A. 0 B. 1 C. 2 D. 3 E. 4

Answer B

Solution Denote the four statements as s_1, s_2, s_3, s_4. Let T and F represent the counts of true and false statements respectively. We know $T + F = 4$.

Assume s_1 is false. This implies $T = 0, F = 4$, which would make s_4 false—a contradiction!

Since at least one of s_1, s_3 (or pick s_4) is true, at least one of s_2, s_4 (or pick s_3) is true, we have $T \geq 2$, confirming s_1, s_2 are true.

If s_4 were false, then $T = 4, F = 0$, contradicting s_4's falsity!

Hence, s_4 is true, yielding $T \geq 3, F \leq 1$, which implies s_3 is false, with $T = 3, F = 1$.

9 Let $f(x) = 100x^3 - 300x^2 + 200x$. For how many real numbers a does the graph of $y = f(x - a)$ pass through the point $(1, 25)$?

A. 1 B. 2 C. 3 D. 4

E. more than 4

Answer C

Solution

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