Each statement is either true or false. How many false statements did Agnes write on the paper?

A. 0 B. 1 C. 2 D. 3 E. 4

Answer B

Solution Denote the four statements as s_1, s_2, s_3, s_4. Let T and F represent the counts of true and false statements respectively. We know $T + F = 4$.

Assume s_1 is false. This implies $T = 0, F = 4$, which would make s_4 false—a contradiction!

Since at least one of s_1, s_3 (or pick s_4) is true, at least one of s_2, s_4 (or pick s_3) is true, we have $T \geq 2$, confirming s_1, s_2 are true.

If s_4 were false, then $T = 4, F = 0$, contradicting s_4's falsity!

Hence, s_4 is true, yielding $T \geq 3, F \leq 1$, which implies s_3 is false, with $T = 3, F = 1$.

9 Let $f(x) = 100x^3 - 300x^2 + 200x$. For how many real numbers a does the graph of $y = f(x - a)$ pass through the point $(1, 25)$?

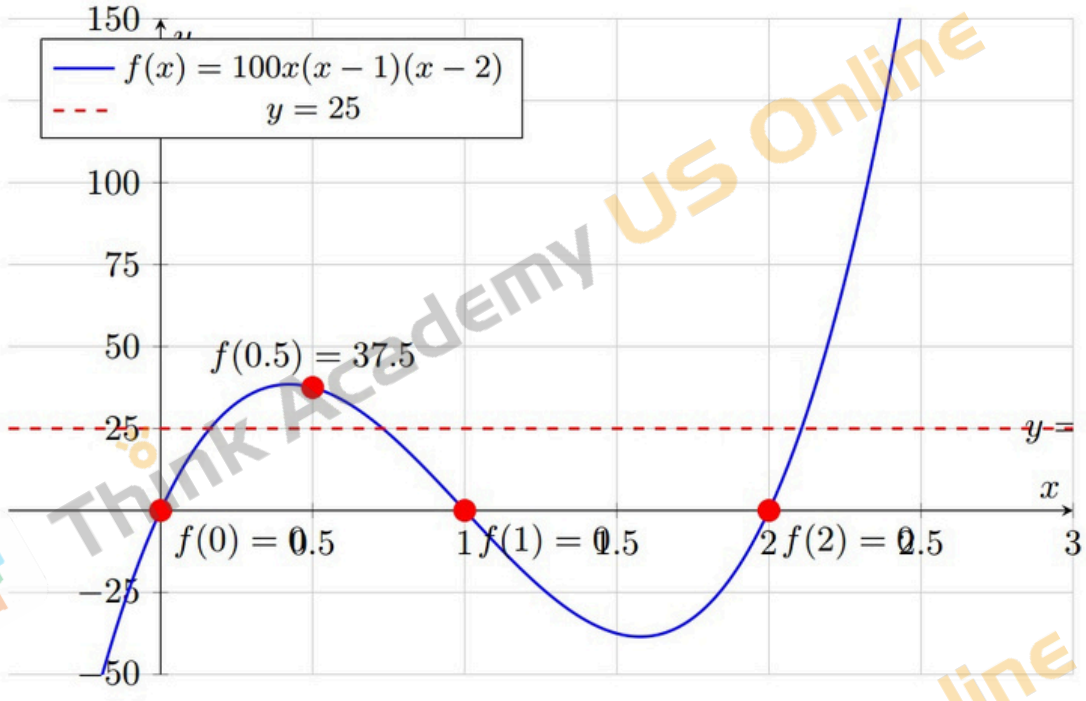
A. 1 B. 2 C. 3 D. 4

E. more than 4

Answer C

Solution

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We seek the number of solutions of $f(1-a)=25$, which is equivalent to the number of solutions of $f(x)=25$.

Since $f(x)=100\times x(x-1)(x-2)$, we have $f(0)=f(1)=f(2)=0$. As the coefficient of x^3 is positive, we can draw the diagram of $f(x)$.

Note that $f\left(\frac{1}{2}\right)=100\times\frac{1}{2}\times\left(-\frac{1}{2}\right)\times\left(-\frac{3}{2}\right)=25\times\frac{3}{2}>25$

By the intermediate value theorem, there exist $x_1\in\left(0,\frac{1}{2}\right)$ and $x_2\in\left(\frac{1}{2},1\right)$ satisfying $f(x_1)=f(x_2)=25$.

Since $f(2)=0$ and $f(3)>100>25$, there exists $x_3\in(2,3)$ with $f(x_3)=25$.

As $f(x)=25$ is a cubic equation with at most 3 real roots, x_1,x_2,x_3 are all the solutions.

10 A semicircle has diameter $\overline{AB}$ and chord $\overline{CD}$ of length 16 parallel to $\overline{AB}$. A smaller semicircle with diameter on $\overline{AB}$ and tangent to $\overline{CD}$ is cut from the larger semicircle, as shown below.

