2025 AMC 10B

1 The instructions on a 350-gram bag of coffee beans say that proper brewing of a large mug of pour-over coffee requires 20 grams of coffee beans. What is the greatest number of properly brewed large mugs of coffee that can be made from the coffee beans in that bag?

A. 16 B. 17 C. 18 D. 19 E. 20

Answer B

Solution $\left\lfloor \frac{350}{20} \right\rfloor = \lfloor 17.5 \rfloor = 17$

2 Jerry wrote down the ones digit of each of the first 2025 positive squares: $1, 4, 9, 6, 5, 6, \cdots$. What is the sum of all the numbers Jerry wrote down?

A. 9025 B. 9070 C. 9090 D. 9115 E. 9160

Answer D

Solution $(10a+b)^2 = 100a^2 + 20ab + b^2 \equiv b^2 \pmod{10}, \quad a, b \in \mathbb{Z}$

Let $f(n)$ be the ones digit of n, then $f(10a+b) = f(b)$.

$\sum_{n=1}^{2025} f(n) = 202 \cdot \sum_{n=1}^{10} f(n) + \sum_{n=1}^{5} f(n)$ which is

$202 \times (1+4+9+6+5+6+9+4+1+0) + (1+4+9+6+5) = 202 \times 45 + 25 = 9115.$

3 A Pascal-like triangle has 10 as the top row and 10 followed by 1 as the second row. In each subsequent row the first number is 10, the last number is 1, and, as in the standard Pascal Triangle, each other number in the row is the sum of the two numbers directly above it. The first four rows are shown below.

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$$\begin{array}{ccccccc} & & & 10 & & & \\ & & 10 & & 1 & & \\ & 10 & & 11 & & 1 & \\ 10 & & 21 & & 12 & & 1 \end{array}$$

What is the sum of the digits of the sum of the numbers in the 11th row?

A. 11 B. 13 C. 14 D. 16 E. 17

Answer D

Solution Let S_k be the sum of k-th row.

Then $S_k = 2 \cdot S_{k-1} - (10+1) + (10+1) = 2S_{k-1}$ for $k \geq 3$.

Since $S_2 = 11$, we have $S_k = 11 \cdot 2^{k-2}$ for $k \geq 2$.

$S_{11} = 11 \times 2^9 = 11 \times 512 = 5632$

4 The value of the two-digit number $\underline{a}\,\underline{b}$ in base seven equals the value of the two-digit number $\underline{b}\,\underline{a}$ in base nine. What is $a+b$?

A. 7 B. 9 C. 10 D. 11 E. 14

Answer A

Solution $(ab)_7 = (ba)_9$ implies $7a + b = 9b + a$ where $0 \leq a, b < 7$

This gives $6a = 8b$, then $3a = 4b$, so $a = 4k$ and $b = 3k$ for $k \in \mathbb{Z}$.

Since $a \in (0, 7)$, we have $k = 1$, thus $a = 4$ and $b = 3$.

5 In $\triangle ABC$, $AB = 10$, $AC = 18$, and $\angle B = 130°$. Let O be the center of the circle containing points A, B, and C. What is the degree measure of $\angle CAO$?

A. 20 B. 30 C. 40 D. 50 E. 60

Answer C

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