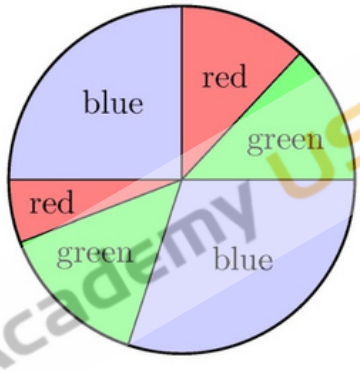
$$\sum_{k=1}^{\infty} \frac{k+4}{k(k+2)(k+4)} = \sum_{k=1}^{\infty} \frac{1}{k(k+2)} = \frac{1}{2} \cdot \left(1 + \frac{1}{2}\right)$$

Therefore:

$$\sum_{k=1}^{\infty} \frac{1}{k(k+2)(k+4)} = \frac{1}{4} \cdot \left(\sum_{k=1}^{\infty} \frac{k+4}{k(k+2)(k+4)} - \sum_{k=1}^{\infty} \frac{k}{k(k+2)(k+4)}\right)$$
$$= \frac{1}{8} \cdot \left(1 + \frac{1}{2} - \frac{1}{3} - \frac{1}{4}\right) = \frac{11}{96}$$

16 A circle has been divided into 6 sectors of 6 different sizes. Then 2 of the sectors are painted red, 2 painted green, and 2 painted blue so that no two neighboring sectors are painted the same color. One such coloring is shown below.



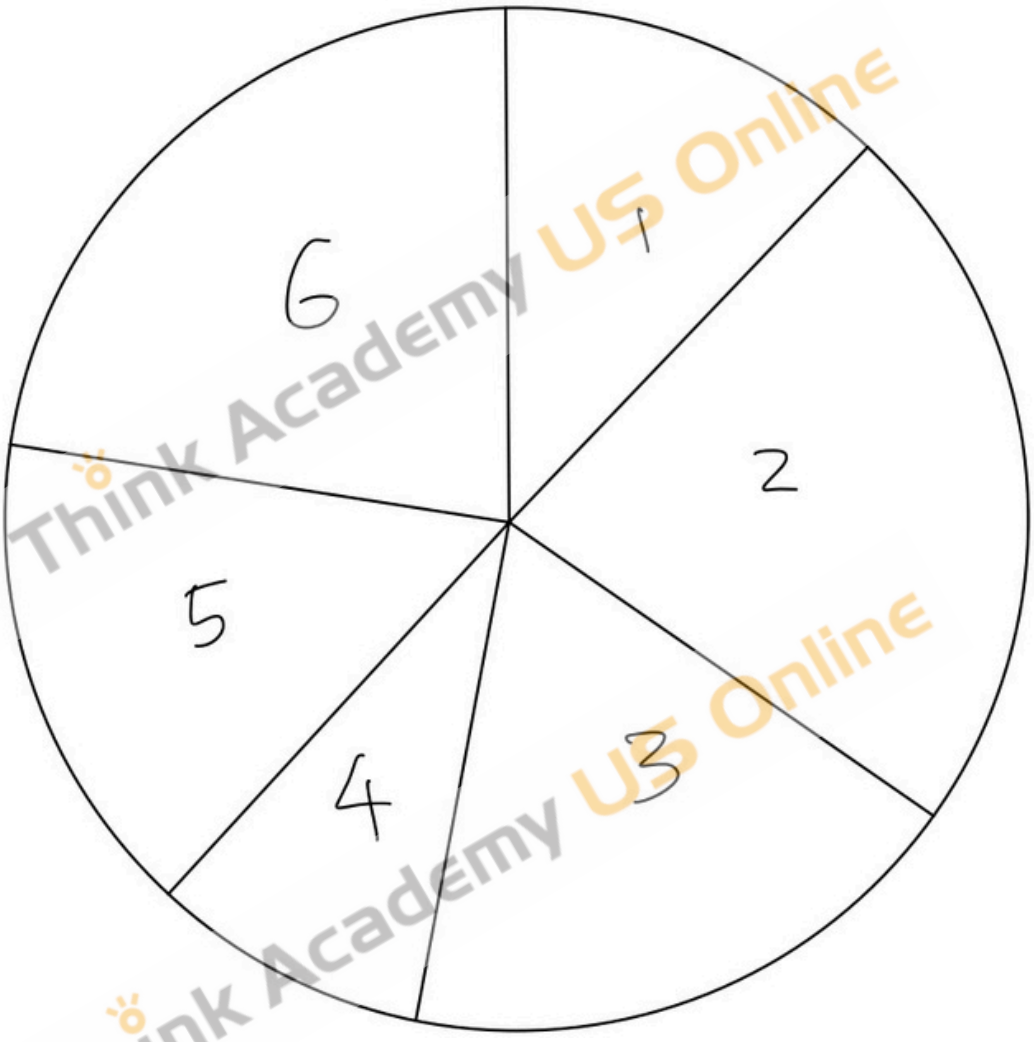
How many different colorings are possible?

A. 12 B. 16 C. 18 D. 24 E. 28

Answer D

Solution

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3 ways to color region 1. When it is red, the other red can only be region 3, 4 or 5.

Case 1: Region 3 is red, then regions 4 and 6 have the same color, regions 2 and 5 have the same. 2 ways of coloring.

Case 2: Region 4 is red, then 2 ways to color regions 2 and 3 with different colors, and likewise for regions 5 and 6. There should be $2 \times 2 = 4$ ways.

Case 3: Region 5 is red, same as Case 1 with 2 ways.

Thus, the total ways $= 3 \times (2 + 4 + 2) = 24$.

17 Consider a decreasing sequence of n positive integers$s x_1 > x_2 > x_3 > \cdots > x_n$ that satisfies the following two conditions:

· The average (arithmetic mean) of the first 3 terms in the sequence is 2025

· For all $4 \leqslant k \leqslant n$ the average of the first k terms in the sequence is 1 less than the average of the first $k - 1$ terms in the sequence.

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What is the greatest possible value of n?

A. 1013 B. 1014 C. 1016 D. 2016 E. 2025

Answer B

Solution Let a_k be the average of the first k terms.

Given $a_3 = 2025$ and $a_k = a_{k-1} - 1$ for $k \geq 4$.

Thus $a_k = 2025 - (k-3) = 2028 - k$ for $k \geq 3$.

$x_k = k \cdot a_k - (k-1) \cdot a_{k-1} = a_k + (k-1) \cdot (a_k - a_{k-1})$

$= 2028 - k - k + 1 = 2029 - 2k$ for $k \geq 4$

For $x_k > 0$, we need $2029 - 2k > 0$, which gives $k \leq 1014$.

18 What is the ones digit of the sum $\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2024}\rfloor + \lfloor\sqrt{2025}\rfloor$?

(Recall that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

A. 1 B. 2 C. 3 D. 5 E. 8

Answer D

Solution $\sum_{n=1}^{2025} \lfloor\sqrt{n}\rfloor = \sum_{\substack{1 \leq n \leq 2025 \\ k^2 \leq n < (k+1)^2}} k$

For $1 \leq k \leq 44$, there are $(k+1)^2 - k^2 = 2k+1$ terms of value k.

For $k = 45$, there is exactly one term.

Thus:

$\sum_{k=1}^{44} k \cdot (2k+1) + 45 = 2\sum_{k=1}^{44} k^2 + \sum_{k=1}^{44} k + 45$

$= 2 \times \frac{1}{6} \times 44 \times 45 \times 89 + \frac{1}{2} \times 44 \times 45 + 45$

Both of the first 2 terms have 0 as ones digit, so the answer is 5.

19 A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate