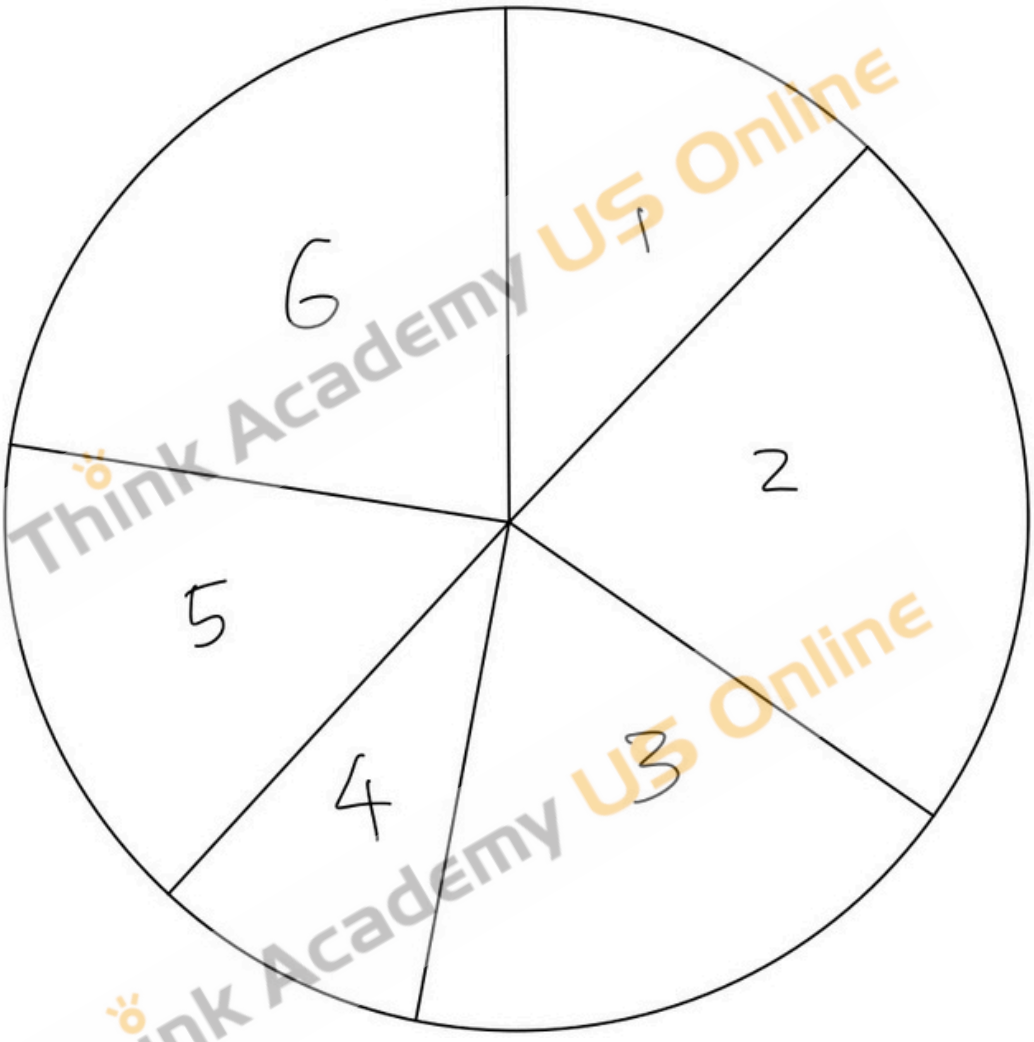


3 ways to color region 1. When it is red, the other red can only be region 3, 4 or 5.

Case 1: Region 3 is red, then regions 4 and 6 have the same color, regions 2 and 5 have the same. 2 ways of coloring.

Case 2: Region 4 is red, then 2 ways to color regions 2 and 3 with different colors, and likewise for regions 5 and 6. There should be $2 \times 2 = 4$ ways.

Case 3: Region 5 is red, same as Case 1 with 2 ways.

Thus, the total ways $= 3 \times (2 + 4 + 2) = 24$.

17 Consider a decreasing sequence of n positive integers$s x_1 > x_2 > x_3 > \cdots > x_n$ that satisfies the following two conditions:

· The average (arithmetic mean) of the first 3 terms in the sequence is 2025

· For all $4 \leqslant k \leqslant n$ the average of the first k terms in the sequence is 1 less than the average of the first $k - 1$ terms in the sequence.

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What is the greatest possible value of n?

A. 1013 B. 1014 C. 1016 D. 2016 E. 2025

Answer B

Solution Let a_k be the average of the first k terms.

Given $a_3 = 2025$ and $a_k = a_{k-1} - 1$ for $k \geq 4$.

Thus $a_k = 2025 - (k-3) = 2028 - k$ for $k \geq 3$.

$x_k = k \cdot a_k - (k-1) \cdot a_{k-1} = a_k + (k-1) \cdot (a_k - a_{k-1})$

$= 2028 - k - k + 1 = 2029 - 2k$ for $k \geq 4$

For $x_k > 0$, we need $2029 - 2k > 0$, which gives $k \leq 1014$.

18 What is the ones digit of the sum $\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2024}\rfloor + \lfloor\sqrt{2025}\rfloor$?

(Recall that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

A. 1 B. 2 C. 3 D. 5 E. 8

Answer D

Solution $\sum_{n=1}^{2025} \lfloor\sqrt{n}\rfloor = \sum_{\substack{1 \leq n \leq 2025 \\ k^2 \leq n < (k+1)^2}} k$

For $1 \leq k \leq 44$, there are $(k+1)^2 - k^2 = 2k+1$ terms of value k.

For $k = 45$, there is exactly one term.

Thus:

$\sum_{k=1}^{44} k \cdot (2k+1) + 45 = 2\sum_{k=1}^{44} k^2 + \sum_{k=1}^{44} k + 45$

$= 2 \times \frac{1}{6} \times 44 \times 45 \times 89 + \frac{1}{2} \times 44 \times 45 + 45$

Both of the first 2 terms have 0 as ones digit, so the answer is 5.

19 A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate