What is the greatest possible value of n?

A. 1013 B. 1014 C. 1016 D. 2016 E. 2025

Answer B

Solution Let a_k be the average of the first k terms.

Given $a_3 = 2025$ and $a_k = a_{k-1} - 1$ for $k \geq 4$.

Thus $a_k = 2025 - (k-3) = 2028 - k$ for $k \geq 3$.

$x_k = k \cdot a_k - (k-1) \cdot a_{k-1} = a_k + (k-1) \cdot (a_k - a_{k-1})$

$= 2028 - k - k + 1 = 2029 - 2k$ for $k \geq 4$

For $x_k > 0$, we need $2029 - 2k > 0$, which gives $k \leq 1014$.

18 What is the ones digit of the sum $\lfloor\sqrt{1}\rfloor + \lfloor\sqrt{2}\rfloor + \lfloor\sqrt{3}\rfloor + \cdots + \lfloor\sqrt{2024}\rfloor + \lfloor\sqrt{2025}\rfloor$?

(Recall that $\lfloor x \rfloor$ denotes the greatest integer less than or equal to x.)

A. 1 B. 2 C. 3 D. 5 E. 8

Answer D

Solution $\sum_{n=1}^{2025} \lfloor\sqrt{n}\rfloor = \sum_{\substack{1 \leq n \leq 2025 \\ k^2 \leq n < (k+1)^2}} k$

For $1 \leq k \leq 44$, there are $(k+1)^2 - k^2 = 2k+1$ terms of value k.

For $k = 45$, there is exactly one term.

Thus:

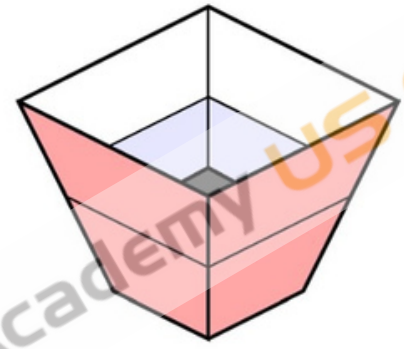
$\sum_{k=1}^{44} k \cdot (2k+1) + 45 = 2\sum_{k=1}^{44} k^2 + \sum_{k=1}^{44} k + 45$

$= 2 \times \frac{1}{6} \times 44 \times 45 \times 89 + \frac{1}{2} \times 44 \times 45 + 45$

Both of the first 2 terms have 0 as ones digit, so the answer is 5.

19 A container has a 1×1 square bottom, a 3×3 open square top, and four congruent trapezoidal sides, as shown. Starting when the container is empty, a hose that runs water at a constant rate

takes 35 minutes to fill the container up to the midline of the trapezoids.



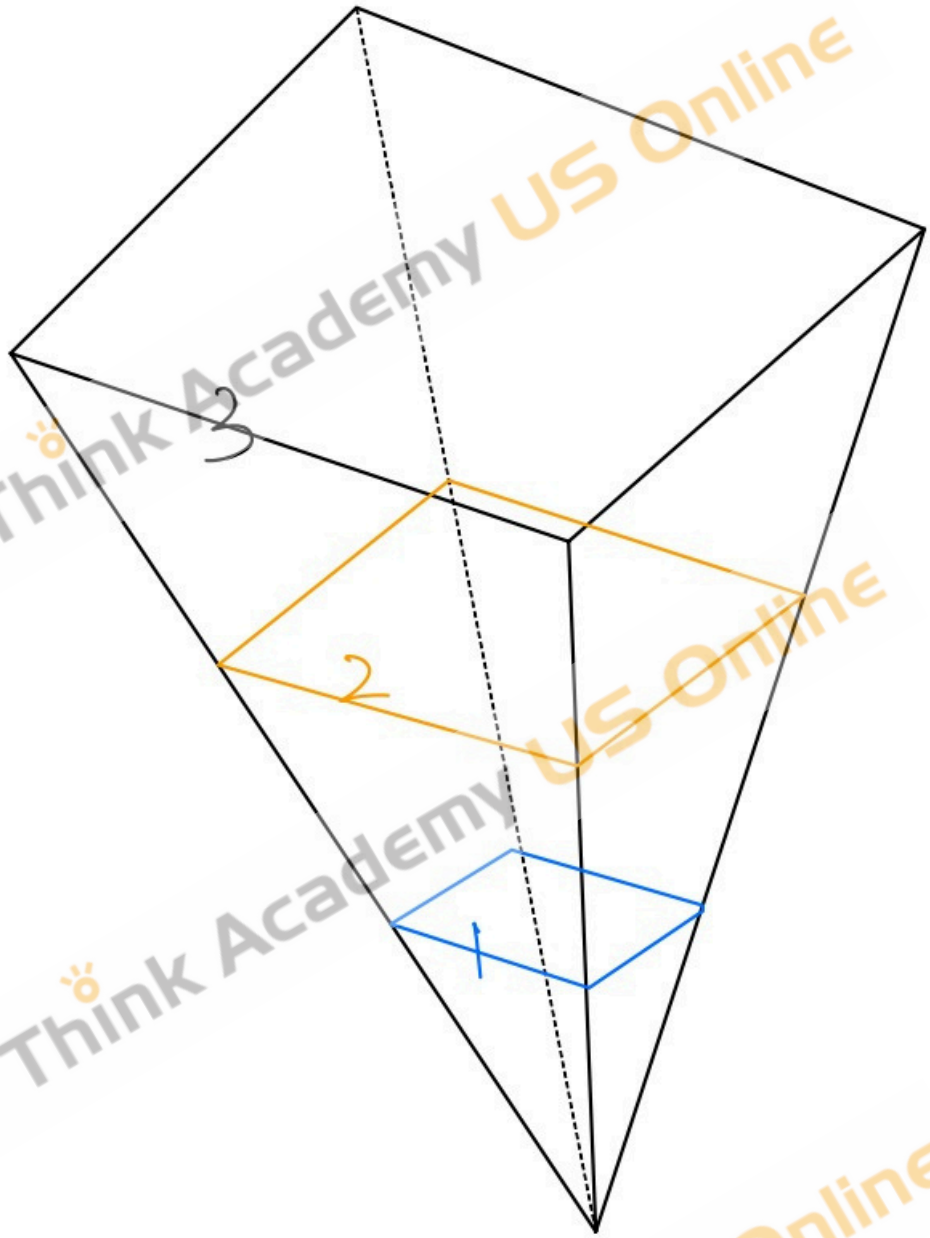
How many more minutes will it take to fill the remainder of the container?

A. 70 B. 85 C. 90 D. 95 E. 105

Answer D

Solution

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Extend the lines so the sides become triangles.

Let h be the height of half the container; then the extended cone also has height h since the side lengths of the squares are 1, 2, 3 respectively.

$$\frac{V_{\text{remained}}}{V_{\text{filled}}} = \frac{\frac{1}{3}\cdot 2h\cdot 2^2 - \frac{1}{3}\cdot 2h\cdot 1^2}{\frac{1}{3}\cdot 2h\cdot 1^2 - \frac{1}{3}\cdot h\cdot 1^2} = \frac{27-8}{8-1} = \frac{19}{7}$$

Therefore, the time $= 35 \times \frac{19}{7} = 95$ mins.

20 Four congruent semicircles are inscribed in a square of side length 1 so that their diameters are on the sides of the square, one endpoint of each diameter is at a vertex of the square, and