# 2026 AMC10B soln

## P1: D

Let $2^{2023} = x$, then the original expression is:

$$\frac{8x + 4x - x}{2x - x} = \boxed{11}$$

## P2: D

The two sculptures are similar with ratio $6cm : 8cm = 3:4$.

So the volume ratio is $(3:4)^3 = 27:64$, and the mass of the larger sculpture is $486g \cdot \frac{64}{27} = \boxed{1152}\,g$

## P3: C

For each of the 8 corners,

the bee can fly within $1/8$ of a sphere with radius $= 1/2$.

Also these "$1/8$ sphere" regions won't overlap with each other.

Thus the bee cannot reach:

$$1 \times 2 \times 3 - 8 \cdot \frac{1}{8} \cdot \frac{4}{3}\pi r^3 = \boxed{6 - \frac{1}{6}\pi} \quad \text{cubic units.}$$

P4: D

Re-group the equation to get:

$$3\left(\frac{1}{x+4}+\frac{1}{x-4}\right) = 4\left(\frac{1}{x+3}+\frac{1}{x-3}\right)$$

$$3\cdot\frac{2x}{x^2-16} = 4\cdot\frac{2x}{x^2-9}$$

Since $x$ is positive and thus non-zero,

$$3\cdot\frac{1}{x^2-16} = 4\cdot\frac{1}{x^2-9}$$

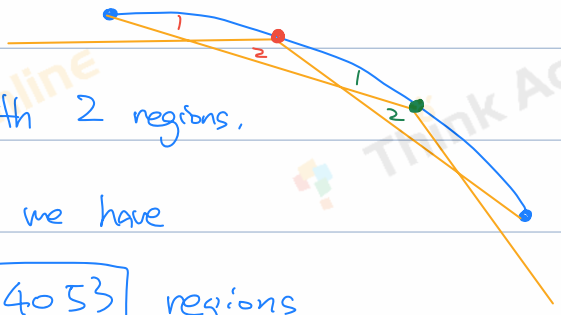
$$3(x^2-9) = 4(x^2-16)$$

$$x^2 = 37$$

$$x = \boxed{\sqrt{37}}$$

P5: B

In the diagram we show part of the circle. We can match each vertex with 2 regions, also there is a central region, so we have




$$2026 \times 2 + 1 = \boxed{4053} \text{ regions.}$$

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P6: C

Lower the degree of the numerator:

$$\frac{n^2-1}{n+2} = \frac{(n^2-4)+3}{n+2} = (n-2) + \frac{3}{n+2} \in \mathbb{Z}.$$

Thus $n+2 \mid 3$, $n+2 = \pm 1, \pm 3$, $n = -5, -3, -1, 1$: 4 soln.

P7: A

Plug in the points into the equation, we have:

$$\begin{cases} c = 0 \\ 9 + 3a + c = 0 \\ 16 + 4b + c = 0 \end{cases}$$

Thus $\begin{cases} a = -3 \\ b = -4 \\ c = 0 \end{cases}$, $a+b+c = -7$.

P8: D

The sequence starting with n has length n. So the sequence will include all same-parity numbers from n to the last number is: $n + 2 \times (n-1) = 3n-2$.

Then 2026 will first appear as the last number: $2026 = 3n-2$,

in the sequence starting with $n = 676$. In the end it will appear in the sequence starting with itself, 2026.

Thus 2026 appears $\frac{2026-676}{2} + 1 = 676$ times, the answer is $6+7+6 = \boxed{19}$.

P9: D

Each new student will choose seat based on current configuration.

We will use the digits of a number to represent which seat is taken.

number of students in blue, and seat configuration in red and green.

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 1 | 13, 14, 15 | 135, 124, 134, 145 | 1235, 1345, 1245, | 12345 |
| 2 | 24, 25 | 245, 125, 235 | | |
| 3 | 35 | | | |
| 4 | | | | |
| 5 | | | | |

There are $\boxed{22}$ configurations.

P10 : E

Simplify the 1st inequality : $-4 \leq x+y \leq 4$

Simplify the 2nd inequality :

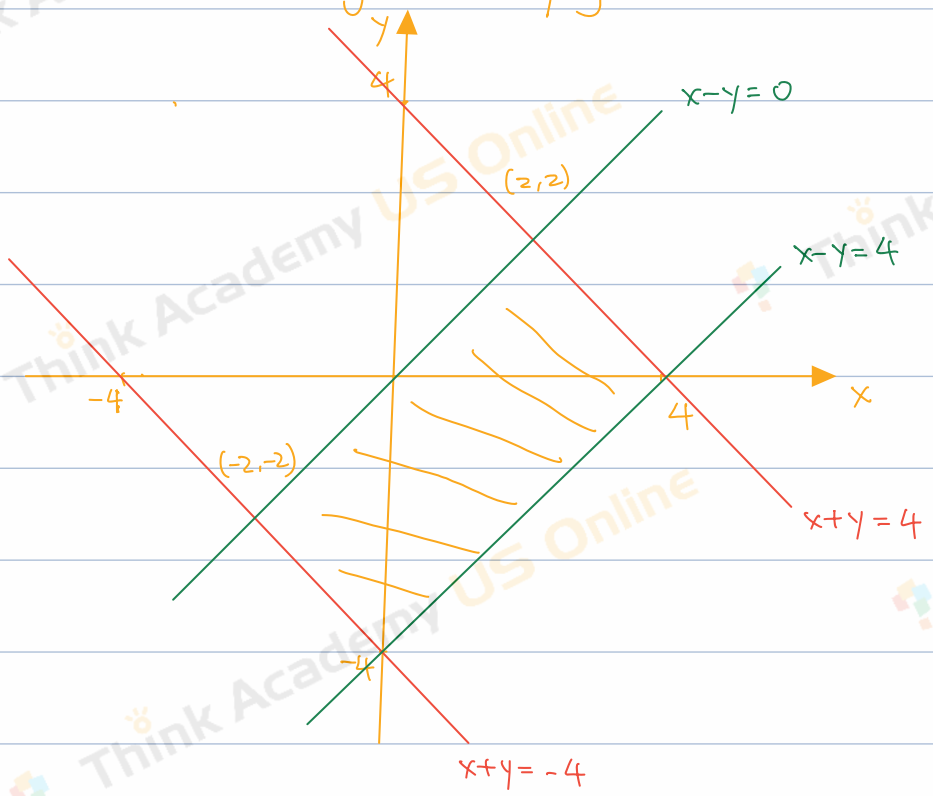
$$x^2+y^2-2xy - 4(x-y) \leq 0$$

$$(x-y)^2 - 4(x-y) \leq 0$$

$$(x-y)(x-y-4) \leq 0$$

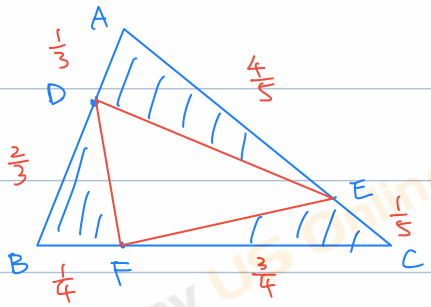
$$0 \leq x-y \leq 4$$

Let's draw the region satisfying the 1st and the 2nd inequality:




So the area is $2\sqrt{2} \times 4\sqrt{2} = \boxed{16}$.

P11 : B




The shaded area ratio is:

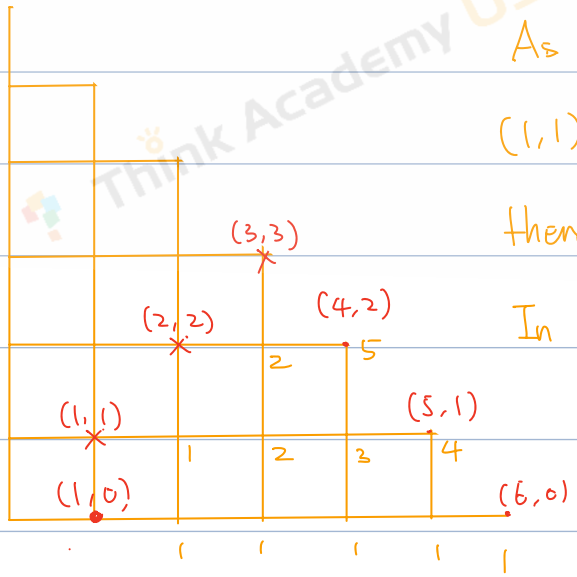
$$\frac{[ADE]+[BFD]+[CEF]}{[ABC]} = \frac{1}{3}\times\frac{4}{5}+\frac{2}{3}\times\frac{1}{4}+\frac{3}{4}\times\frac{1}{5} = \frac{35}{60} = \frac{7}{12}$$

Thus $[DEF]/[ABC] = 1 - 7/12 = \boxed{\frac{5}{12}}$

P12 : D

Let's move 1 unit right for head, or 1 unit up for tail.

We are at (1,0) now.




As in the diagram, without reaching

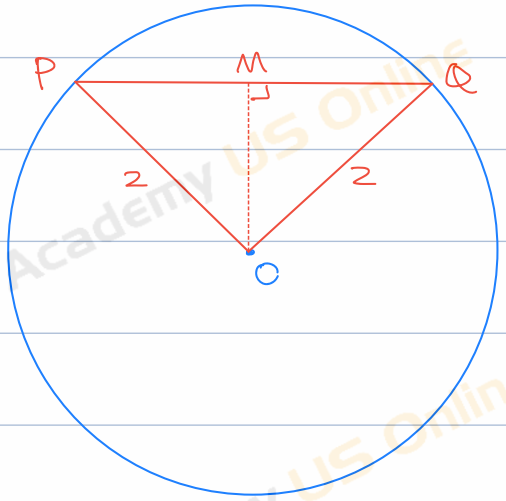
(1,1), (2,2), (3,3),

there are $5+4+1 = 10$ ways.

In total, there are $2^5 = 32$ ways.

So the probability is $\frac{10}{32} = \boxed{\frac{5}{16}}$

P13 : E




The radius is $\sqrt{4} = 2$.

Let M be the midpoint of PQ.

So $OM \perp PQ$, $OM \geq 1$.

Then $\angle POM \leq 60°$,

$\angle POQ = 2\angle POM \leq 120°$.

Assume we pick P first, then Q, the probability of $\angle POQ \leq 120°$ is: $\frac{2 \times 120°}{360°} = \boxed{\frac{2}{3}}$.

P14 : E

Let $x = 2^r$, $y = 2^s$

We have $\begin{cases} x + y = 7 \\ xy = 2^{r+s} = 4 \end{cases}$

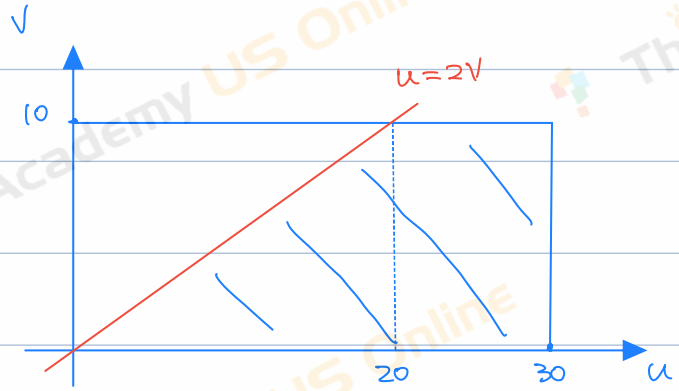
Then $8^r + 8^s = x^3 + y^3 = (x+y)^3 - 3xy(x+y)$

$= 7^3 - 3 \times 4 \times 7 = \boxed{259}$

P15: D

The quadratic equation has a real solution if

$$\Delta = u^2 - 4v^2 \geq 0, \text{ so } |u| \geq 2|v|.$$




As in the diagram, the probability is $\frac{\text{desired area}}{\text{total area}} = \boxed{\frac{2}{3}}$

P16: E

Case 1: single-color grid

There are 3 ways to color the grid with one color.

Case 2: 2-color grid

We may have white-red or white-blue grid.

So there are $2 \times (2^4 - 2) = 28$ ways

($2^4$: 2 choices for 4 squares; $-2$: exclude single-color grid)