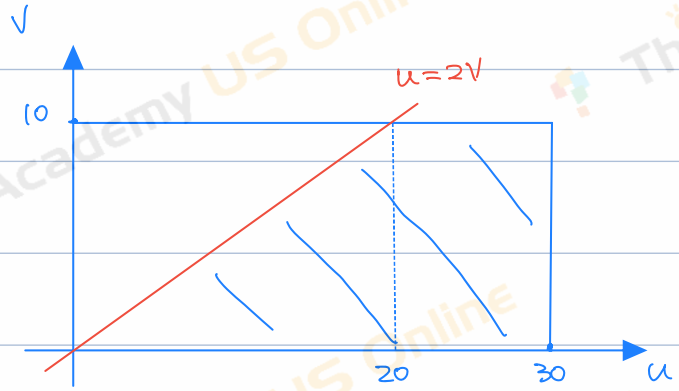
P15: D

The quadratic equation has a real solution if

$$\Delta = u^2 - 4v^2 \geq 0, \text{ so } |u| \geq 2|v|.$$



As in the diagram, the probability is $\frac{\text{desired area}}{\text{total area}} = \boxed{\frac{2}{3}}$

P16: E

Case 1: single-color grid

There are 3 ways to color the grid with one color.

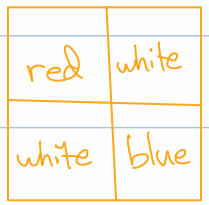
Case 2: 2-color grid

We may have white-red or white-blue grid.

So there are $2 \times (2^4 - 2) = 28$ ways

(2^4: 2 choices for 4 squares; -2: exclude single-color grid)

Case 3: 3-color grid



The only possibility is one red, one blue square at opposite corners.

There are $2 \times 2 = 4$ ways.

In total, there are $3 + 28 + 4 = \boxed{35}$ ways.

P17: B

Suppose there are n false statements, $0 \leq n \leq 2026$.

Case 1: $n < 1001$

Then all the statements are false, $n = 2026$, contradiction.

Case 2: $1001 \leq n \leq 2026$

So the $(n-1000)$th statement is "there are at least n false statements." Thus there are $n-1000$ true statements.

We have $n + (n-1000) = 2026$, $n = 1513$.

So there are $n - 1000 = \boxed{513}$ true statements.

P18: C

Factorize $2025 = 3^4 \times 5^2$

Within 2026!, there are 1010 factors of 3, as shown below.

$$\left\lfloor \frac{2026}{3} \right\rfloor = 675$$
$$\left\lfloor \frac{675}{3} \right\rfloor = 225$$
$$\left\lfloor \frac{225}{3} \right\rfloor = 75$$
$$\left\lfloor \frac{75}{3} \right\rfloor = 25$$
$$\left\lfloor \frac{25}{3} \right\rfloor = 8$$
$$\left\lfloor \frac{8}{3} \right\rfloor = 2$$

sum = 1010

There are 505 factors of 5, as shown below.

$$\left\lfloor \frac{2026}{5} \right\rfloor = 405$$
$$\left\lfloor \frac{405}{5} \right\rfloor = 81$$
$$\left\lfloor \frac{81}{5} \right\rfloor = 16$$
$$\left\lfloor \frac{16}{5} \right\rfloor = 3$$

sum = 505

Thus $n = \left\lfloor \frac{1010}{4} \right\rfloor + 1 = \left\lfloor \frac{505}{2} \right\rfloor + 1 = \boxed{253}$.

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P19: D

If Anita goes first,

① at $\frac{12}{20} = \frac{3}{5}$ probability, she draws green and win.

② at $\frac{8}{20} = \frac{2}{5}$ probability, she draws red. As they draw with replacement, the situation is equivalent to "Ben goes first".

So Ben's winning probability in this situation is:

$$A = \frac{2}{5} \cdot B$$

If Ben goes first,

① at $\frac{8}{20} = \frac{2}{5}$ probability, he draws red and win.

② at $\frac{12}{20} = \frac{3}{5}$ probability, he draws green. As they draw with replacement, the situation is equivalent to "Anita goes first".

So his winning probability in this situation is:

$$B = \frac{2}{5} + \frac{3}{5} A$$

Solving for A, B, we have $\begin{cases} A = \frac{4}{19} \\ B = \frac{10}{19} \end{cases}$

$$B - A = \boxed{\frac{6}{19}}.$$

P20 : E

Let their integers be A, B, C respectively.

List the possible outcomes of A+B:

A+B = S	2	3	4	.....	10	11	12	.....	20
ways	1	2	3	...	9	10	9	....	1

S-1 | 21-S

Consider A+B mod 7

A+B (mod 7)	0	1	2	3	4	5	6
A+B	7,14	8,15	2,9,16	.....			
ways	13	13	14	15	16	15	14

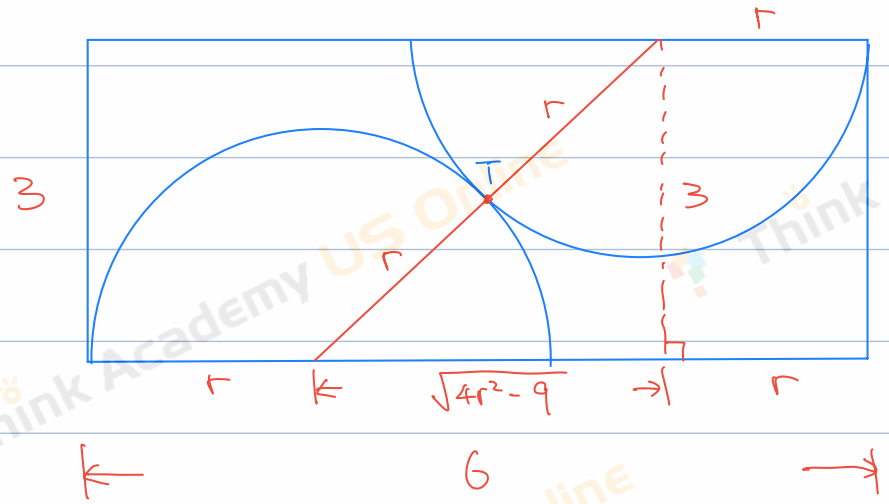
Consider C mod 7

C (mod 7)	0	1	2	3	4	5	6
ways	1	2	2	2	1	1	1

So there are $13\times1+13\times1+14\times1+15\times1+16\times2+15\times2+14\times2$

$= \boxed{145}$ ways to make $7 \mid A+B+C$.

P21: C



Connect the circle centers and pass through T.

As in the diagram, we have $2r+\sqrt{4r^2-9}=6$

$$\sqrt{4r^2-9}=6-2r$$

$$4r^2-9=4r^2-24r+36$$

$$r=\boxed{\frac{15}{8}}$$

P22: C

For a dignified path, we can completely fix it by:

① "width": the value of $b-a$,

② "height at each step": when Belinda moves right, what is her y-coordinate?

There are $(b-a)$ such coordinates and they can be $1\sim4$.

③ "horizontal location": the value of a.

There are $(5-(b-a))$ such choices.

So in total, there are $\sum_{b-a=1}^{4} 4^{(b-a)} \cdot (5-(b-a))$

$= 4^1 \cdot 4 + 4^2 \cdot 3 + 4^3 \cdot 2 + 4^4 \cdot 1$

$= \boxed{448}$ dignified paths.

P23: D

Call a face with 2 or more blue vertices as a "blue face".

Consider a single face.

For 0 blue vertices, there are $2^4 = 16$ ways

For 1 blue vertex, there are $4 \times 2^3 = 32$ ways

In total there are $3^4 = 81$ ways to color.

So E(number of blue face among a single face) $= 1 - \frac{16+32}{81} = \frac{11}{27}$

Thus E(number of blue face among six faces)

$= 6 \cdot$ E(number of blue face among a single face)

$= \boxed{\frac{22}{9}}$

P24: C

Let the sequence be $\{a_n\}$.

So $a_{n+3} = \frac{1}{3}(a_n + a_{n+1} + a_{n+2})$

$3a_{n+3} = a_n + a_{n+1} + a_{n+2}$

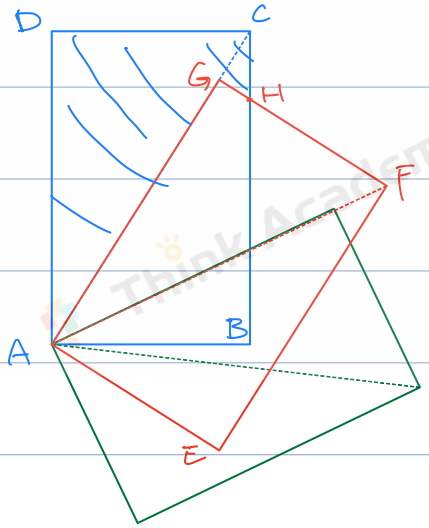
$a_{n+1} + 2a_{n+2} + 3a_{n+3} = a_n + 2a_{n+2} + 3a_{n+3}$, which is invariant of n.

As we are averaging the terms, we can assume the sequence will converge to a limit L. When the index is large, such as 2026, we have: $a_{2026} \approx a_{2027} \approx a_{2028} \approx L$

So $6a_{2026} \approx a_{2026} + 2a_{2027} + 3a_{2028} = a_1 + 2a_2 + 3a_3$

$\boxed{(3,4,8)}$ gives the largest $a_1 + 2a_2 + 3a_3$.

P25: A



Since $\frac{AC}{AB} = 2$, $\angle DAC = 30°$

So there are $\frac{360°}{30°} = 12$ cards in total.

Focus on the shaded ADCHG, each card contributes this area.

As $AG = AD = \sqrt{3}$, $CG = AC - AG = 2 - \sqrt{3}$.

Since Rt$\triangle$CGH $\sim$ Rt$\triangle$ADC, $GH = \frac{CG}{\sqrt{3}} = \frac{2\sqrt{3}}{3} - 1$

So $[ADCHG] = [ADC] + [CGH]$

$$= \frac{1}{2} \times 1 \times \sqrt{3} + \frac{1}{2} \times (2-\sqrt{3})\left(\frac{2\sqrt{3}}{3} - 1\right)$$

$$= \frac{5\sqrt{3}}{3} - 2$$

The total area is $12[ADCHG] = \boxed{20\sqrt{3} - 24}$

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