AMC 12 A Solution

P1: A

without tax : 100% tax : 27%

total price : 127%

So the percentage is $\frac{27}{127} = 21.2\% \approx \boxed{21\%}$

P2: D

$\because \frac{AF}{FE} = \frac{AD}{BE} = 2$

$\therefore [BEF] = \frac{1}{3}[ABE] = \frac{1}{12}[ABCD] = \boxed{48}$

P3: A

Assume that it is a n-gon.

Then the sum of internal angles is : $(n-2)\cdot 180^\circ$,

which is also $\frac{n}{2}\cdot 150^\circ + \frac{n}{2}\cdot 190^\circ$.

Thus $n = \boxed{36}$

P4: C

Let the average of the first n terms be $A_n = \frac{a_1+a_2+\cdots+a_n}{n}$.

For $n \geq 2$, we have $n+1 \geq 3$, $a_{n+1} = A_n$.

Thus $A_{n+1} = \frac{(a_1+a_2+\cdots+a_n)+a_{n+1}}{n+1} = \frac{(n\cdot A_n)+A_n}{n+1} = A_n$.

So $a_{10} = A_9 = A_8 = \cdots = A_2 = \boxed{142}$

P5: A

Mark the expressions as ①, ② respectively.

So $y+z = \frac{②-①}{2} = 5$

$x = ① - (y+z) = 26$

There are $\boxed{4}$ solutions of y, z whereas x is fixed.

P6: C

The mean is $\frac{1^2+2^2+\cdots+n^2}{n} = \left[\frac{1}{6}n(n+1)(2n+1)\right]/n = {}^{(n+1)(2n+1)}/_{6}$

The median for odd n is: $\left(\frac{n+1}{2}\right)^2$

So $\frac{1}{6}(n+1)(2n+1) = (1+32\%)\left(\frac{n+1}{2}\right)^2$

$\Longrightarrow \quad n = \boxed{49}$

P7: A

We have $1+i = \sqrt{2}e^{i\frac{\pi}{4}}$

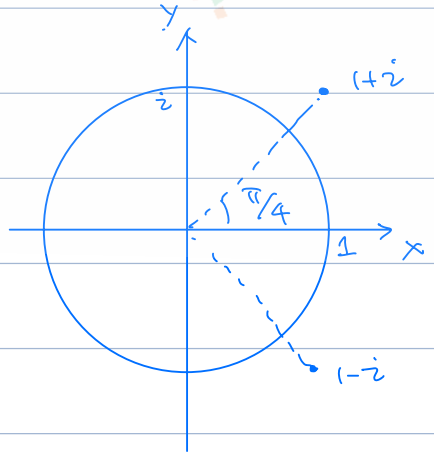
$1-i = \sqrt{2}e^{-i\frac{\pi}{4}}$

Thus $(1+i)^{20} = \sqrt{2}^{20} \cdot e^{i\frac{\pi}{4}\times 20}$

$= 2^{10} \cdot e^{i(5\pi)} = -2^{10}$

$(1-i)^{20} = \overline{(1+i)^{20}} = -2^{10}$

Their sum is $-2^{11} = \boxed{-2048}$



P8: C

	AMC10	AMC12
registrant	25	75
show-up rate	x	y
participant	$25x$	$75y$

So we want to minimize $25x + 75y$ when $\frac{x+y}{2} = 60\%$.

Let $x = 100\%$, $y = 20\%$, we have

$25 \times 100\% + 75 \times 20\% = \boxed{40}$

P9: E

When $\frac{16x}{y} = xy$,

Case 1: $x = 0$.

Then $|x^2 - y^2| = 4$ gives $y = \pm 2$. $\Rightarrow$ 2 soln.

Case 2: $x \neq 0$, so $\frac{16}{y} = y$.

Then $y^2 = 16$, $y = \pm 4$, 2 choices

$x^2 = 12$ or 20, $x = \pm\sqrt{12}$ or $\pm\sqrt{20}$. 4 choices.

$\Rightarrow 2 \times 4 = 8$ soln.

Finally we have $2 + 8 = \boxed{10}$ soln.

P10 : B

Tracing backwards, if $\frac{a}{b}$'s next level is $\frac{20}{27}$,

then $\frac{20}{27} = \frac{a}{a+b}$ or $\frac{b}{a+b} \Longrightarrow \begin{cases} a = 20, 7 \\ b = 7, 20. \end{cases}$

Since every fraction on the tree is either the top $\frac{1}{2}$, or generated as $\frac{a}{a+b}, \frac{b}{a+b}$, all of them are in $(0,1)$.

Thus it can only be $\frac{7}{20}$ that generated $\frac{20}{27}$.

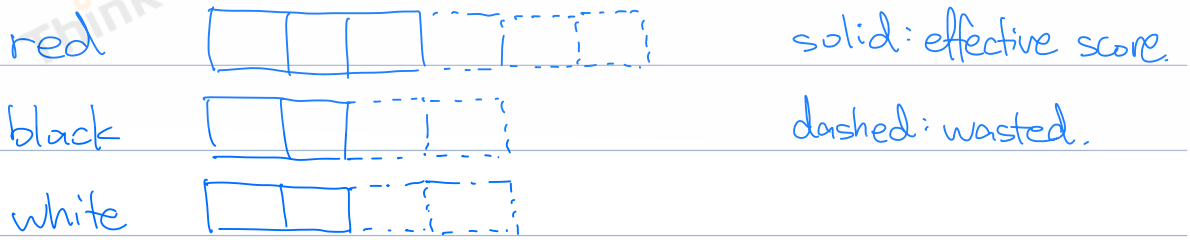
Repeat the tracing backwards:

$$\frac{20}{27} \leftarrow \frac{7}{20} \leftarrow \frac{7}{13} \leftarrow \frac{6}{7} \leftarrow \frac{1}{6} \leftarrow \frac{1}{5}$$

$$\leftarrow \frac{1}{4} \leftarrow \frac{1}{3} \leftarrow \frac{1}{2} \text{ (level 0)}.$$

So $\frac{20}{27}$ is at level $\boxed{8}$.

P11 : D



Notice that the minimum score is 4, with possible

score counts (3, 1, 0) (red, black, white), (3, 0, 1) and (0, 2, 2);

ball counts (6, 1, 0), (6, 0, 1) and (0, 4, 3)

(0, 3, 4).

$\Rightarrow$ number of ways to get lowest score:

$$\binom{6}{6}\binom{4}{1} + \binom{6}{6}\binom{4}{1} + \binom{4}{4}\binom{4}{3} + \binom{4}{3}\binom{4}{4} = 16$$

number of total ways to choose 7 balls out of 6+4+4=14 balls:

$$\binom{14}{7} = 3432.$$

The probability is $16/3432 = \boxed{2/429}$

P12: E

Since lcm(2,3,4,6,12) = 12, we only need to consider modulo 12. We will list which condition x satisfies:

$x \equiv 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 \pmod{12}$

1st 3rd 1st 2nd 1st 3rd 1st 5th 1st 2nd 1st 4th

So all $1 \sim 2026$ satisfy at least one condition.

The answer is $\boxed{2026}$.

P13: C

Every increasing sequence $a<b<c<d$ has one-to-one correspondence with a 4-number set $\{a,b,c,d\}$, thus there are $\binom{8}{4}$ ways to pick 4 numbers from 1~8, and generate an increasing $a<b<c<d$.

On the other hand, there are 8^4 ways to generate a sequence.

So the probability is $\binom{8}{4} / 8^4 = \boxed{35/2048}$

P14: C

The space inside 2 boxes is: $(2a-2) \times (b-2) \times (c-2)$,

which can contain (without spare room): $a \times b \times c$

Since $a<b<c$, $b-2 < b, c$. Thus $b-2=a$, $c-2=b$, $2a-2=c$.

$\Rightarrow a=6, b=8, c=10$, with a sum $6+8+10=\boxed{24}$

P15: A

We have $\sin(a+b)\sin(a-b) = \frac{1}{2}\left[\left(\cos\left((a+b)-(a-b)\right)-\cos\left((a+b)+(a-b)\right)\right]\right.$

$= \frac{1}{2}(\cos 2b - \cos 2a)$

$= \frac{1}{2}\left[(1-2\sin^2 b)-(1-2\sin^2 a)\right]$

$= \sin^2 a - \sin^2 b = \frac{3}{64}$.

Thus $\log_2\left(\frac{3}{64}\right) = \log_2 3 - \log_2 64 = \log_2 3 - 6$

As $\log_2 3 \in (1,2)$, the final value $\in \boxed{(-5}, -4)$.

P16: C.

Consider how many "a" squares is in the section of our central square.

b	a_2	b
a_1		a_3
b	a_4	b

Case 1: two "a". There are 2 legit sections:

a_1a_3 or a_2a_4.

Case 2: one "a", then there is also one "b" in the partition. We have

4 choices for "a" × 2 choices for "b"

= 8 legit sections.

After the section of the central square is determined, the whole partition is also determined, so we have 10 partitions.